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TECHNICAL NOTE 3809

A METHOD FOR CALCULATION OF FREE-SPACE SOUND PRESSURES  
NEAR A PROPELLER IN FLIGHT INCLUDING CONSIDERATIONS  
OF THE CHORDWISE BLADE LOADING

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## SUMMARY

The general expressions of NACA Report 1198 for determining the sound-pressure field for a rotating propeller in uniform subsonic flight are reviewed for the case of a propeller with uniform chordwise forces. Consideration is given to effects of nonuniform chordwise and radial blade loading. Tabulated values of certain definite integrals that are involved in the calculation of a near-field propeller noise regardless of the form of the chordwise forces are presented. These tabulations cover a wide range of operating conditions and are useful for estimating propeller noise when either the concept of an effective radius or radial distributions of forces are considered. In order to illustrate their use, the tabulations are used in conjunction with the concept of an effective radius to make evaluations for some specific propellers. Results of these evaluations are presented and discussed.

## INTRODUCTION

In references 1 and 2 the sound-pressure field of an "on-stand" propeller is analyzed by replacing the normal-pressure distributions associated with thrust and torque over the propeller by distributions of pressure doublets acting at the propeller disk. In reference 3 the analyses of references 1 and 2 are extended to the case of an in-flight propeller by considering the pressure doublets that represent the thrust and torque to be subjected to a uniform rectilinear motion.

In references 1 and 3 the equations for the sound pressure are derived in exact form, that is, within the realm of linearized potential theory. For convenience in calculation, however, these equations are ultimately simplified so that they pertain only to the first few harmonics of a propeller of low solidity. These simplifications involve the assumptions that the radial load distribution on a propeller blade can be considered as concentrated at some effective radial position and that the

propeller blade chord is so small that the chordwise load at any radial station can be considered as having the form of an impulse or Dirac delta function. Calculations in references 1 to 3 are based on these assumptions.

The present report begins with the general equations that are derived in reference 3 for the sound field of an in-flight propeller and reviews the specialization of these equations to the case of a uniform or rectangular-type chordwise loading. It is deduced from results for this case that, as long as the chordwise loading remains unchanged throughout the propeller cycle, certain definite integrals involved in the expressions for the sound field are the same regardless of the form of the chordwise loading. In the case of the "far field," or for field points located at distances of several propeller diameters from the propeller, the definite integrals can be evaluated in terms of Bessel functions of the first kind, but, in the case of the "near field" or for field points located within a distance of two or three propeller diameters from the propeller, the integrals must be evaluated by numerical procedures. These numerical evaluations for the near-field case can be made once and for all, however, so as to apply to any chordwise loading that remains unchanged throughout the propeller cycle. A main purpose of this report is therefore to present tabulated values of these definite integrals for a range of parameters that correspond to a large range of operating conditions so that the sound field of a given propeller can be fairly rapidly estimated.

Other purposes of the report are: (a) to consider the effects on the sound field of nonuniform types of chordwise blade loading, (b) to consider a method of evaluating expressions for the sound field when radial distributions of loading are taken into account, and (c) to present and discuss results of some calculations, based on the tabulated integrals, of the sound pressures for some specific propellers. The reduction of definite integrals pertaining to the far-field case to Bessel functions of the first kind and a Fourier series development of some simple shapes that are useful for representing nonuniform chordwise force distributions are presented in the appendixes.

### SYMBOLS

$B$	number of propeller blades
$b(r)$	width of propeller blades or blade chord
$c$	velocity of sound
$c_n$	section normal-force coefficient

D propeller diameter

$F(r, \theta, t)$  chordwise distribution of forces perpendicular to thrust of propeller blade and giving rise to torque

$G_1, G_2$  functions defined in equations (31) and (32)

$\Pi_v, I_v, I_v', I_v''$  functions defined in equations (13), (15), (18), and (19)

$J_n$  Bessel function of first kind with index  $n$

$$K_1 = \frac{2\beta^2 r y}{x^2 + \beta^2 y^2 + \beta^2 r^2}$$

$$K_2 = \frac{\Omega}{\beta^2 c} \sqrt{x^2 + \beta^2 y^2 + \beta^2 r^2}$$

$$k = \omega/c$$

M Mach number,  $V/c$

$M_F$  flight Mach number

$M_R$  rotational Mach number

$M_t$  tip Mach number

$m$  order of harmonic

$n$  propeller rotational speed

$p_T$  pressure due to thrust

$p_Q$  pressure due to torque

$p_{rms}$  root-mean-square pressure

$Q$  torque

$R$  length (radius) of propeller blades

$R_e$  effective (radius) length of propeller blades

R.P.      real part of any expression  
 r          radius to blade element  
 r,  $\theta$       polar coordinates in yz-plane

$$S = \sqrt{x^2 + \beta^2 y^2 + \beta^2 r^2 - 2\beta^2 ry \cos \theta}$$

T          thrust  
 t          time  
 t'        dummy variable  
 V          forward velocity  
 $V_F$       forward flight velocity  
 $V_t$       tip velocity

x, y, z,  $y_1, z_1$       Cartesian coordinates

$$\beta = \sqrt{1 - M^2}$$

$\beta_{0.75R}$       blade angle at 0.75 tip radius

$\mu_1, \mu_2$       constants

$\eta$           propeller efficiency

$$\sigma = \frac{Mx + S}{\beta^2}$$

$$\tau = b/r\Omega$$

$\tau_0$         period,  $2\pi/B\Omega$

$\Omega$           angular velocity

$\omega$           frequency of mth harmonic,  $mB\Omega$

Subscripts:

j          jth element  
 lt        left triangle

r        rectangle  
 rt       right triangle  
 t        triangle  
 v = 1, 2, and 3

## ANALYSIS

The sound-pressure equations for chordwise rectangular loading are derived and reduced to expressions involving certain definite integrals denoted by  $I_v$  ( $v = 1, 2$ , and  $3$ ). It is deduced that the integrals  $I_v$  are common to propeller-noise theory; hence, they are evaluated and tabulated over useful ranges of certain key parameters. This tabulation is discussed briefly and then consideration is given to effects of other types of chordwise loading and to a method of approximating a radial integration involved in the sound-pressure equations.

### Sound-Pressure Equations for Chordwise Rectangular

#### Blade Loading

Although the main purpose of this report is to present tables of certain definite integrals that arise in expressions for the near sound field of a propeller, it is desirable, for the sake of completeness and discussion, to derive the integrals by specializing the general expression for the sound field to some particular form of chordwise loading. For this purpose the general equations derived in reference 3 are specialized to the case of rectangular chordwise loading. More general types of loading are discussed in a subsequent section.

From equations (6) to (12) of reference 3, the equations for the sound pressure  $p_T$  associated with a thrust distribution  $F_1(t, r, \theta)$  and the sound pressure  $p_Q$  associated with a torque distribution  $F_2(t, r, \theta)$ ,  $F_1$  and  $F_2$  being periodic with respect to time, can be written as (see fig. 1 for illustration of coordinate system):

$$p_T = -\frac{1}{4\pi} \int_0^R \int_0^{2\pi} F_1(t, r, \theta) \frac{\partial}{\partial x} \frac{e^{-ikr}}{r} dr d\theta \quad (1)$$

and

$$p_Q = \frac{1}{4\pi} \int_0^R \int_0^{2\pi} F_2(t, r, \theta) \frac{1}{r} \frac{\partial}{\partial \theta} \frac{e^{-ikr\sigma}}{S} dr d\theta \quad (2)$$

In these equations  $R$  is radius,  $r$  is the blade element radius,  $t$  is time,  $\theta$  is a polar angle in the  $yz$ -plane,  $\beta = \sqrt{1 - M_F^2}$ ,  $S = \sqrt{x^2 + \beta^2 y^2 + \beta^2 r^2 - \beta^2 r y \cos \theta}$ , and  $\sigma = \frac{Mx + S}{\beta^2}$ ;  $k = \frac{\omega}{c} = \frac{mB\Omega}{c}$  is a frequency parameter involving order of harmonic  $m$ , number of blades  $B$ , angular velocity  $\Omega$ , and velocity of sound  $c$ . It is to be noted that the expressions for  $p_T$  and  $p_Q$  (eqs. (1) and (2)) have been made independent of the coordinate  $z$ . This is a convenient simplification that follows from rotational symmetry when the loading is assumed to remain unchanged throughout the propeller cycle.

When the periodic functions  $F_1$  and  $F_2$  are known, they can be expressed in the form of Fourier series that enables one to treat each harmonic of the associated noise spectra separately. When, for simplicity, the forces acting on the propeller are assumed to be uniformly distributed at each radial station of the propeller blade, that is, when the chordwise distributions of forces are assumed to be rectangular, the functions  $F_1(t, r, \theta)$  and  $F_2(t, r, \theta)$  may be expressed, first for  $\theta = 0$ , as (see ref. 3)

$$\left. \begin{aligned} F_1(t, r, 0) &= \frac{1}{B} \frac{dT}{dr} \frac{r}{b} \frac{d\theta}{dr} dr & (0 < t < \tau) \\ F_1(t, r, 0) &= 0 & (\tau < t < \tau_0) \end{aligned} \right\} \quad (3)$$

$$\left. \begin{aligned} F_2(t, r, 0) &= \frac{1}{B} \frac{1}{r} \frac{dQ}{dr} \frac{r}{b} \frac{d\theta}{dr} dr & (0 < t < \tau) \\ F_2(t, r, 0) &= 0 & (\tau < t < \tau_0) \end{aligned} \right\} \quad (4)$$

where  $T$  is thrust,  $b$  is blade chord,  $Q$  is torque,  $\tau = \frac{b}{r\Omega}$  is the time elapsed for a blade chord with angular velocity  $\Omega$  to pass a given point, and  $\tau_0 = \frac{2\pi}{B\Omega}$  is time elapsed for corresponding parts (for example, the leading edges) of two consecutive blades to pass a given point.

The expressions for  $F_1(t, r, 0)$  and  $F_2(t, r, 0)$  in equations (3) and (4), respectively, may be harmonically analyzed to yield

$$\begin{aligned}
 F_1(t, r, 0) &= \frac{r}{bB\tau_0} \frac{dT}{dr} d\theta dr \sum_{m=-\infty}^{\infty} \int_0^{\tau} e^{\frac{2m\pi i}{\tau_0}(t-t')} dt' \\
 &= \frac{1}{\pi} \sum_{-\infty}^{\infty} \frac{r}{mbB} \left( \sin \frac{mbB}{2r} \right) e^{imB\Omega \left( t - \frac{b}{2\Omega r} \right)} \frac{dT}{dr} dr d\theta
 \end{aligned} \quad (5)$$

and

$$F_2(t, r, 0) = \frac{1}{\pi} \sum_{-\infty}^{\infty} \frac{r}{mbB} \left( \sin \frac{mbB}{2r} \right) e^{imB\Omega \left( t - \frac{b}{2\Omega r} \right)} \frac{1}{r} \frac{dQ}{dr} dr d\theta \quad (6)$$

In these expressions the zeroth harmonic,  $m = 0$ , corresponds to the instantaneous average thrust and torque over the blade element which does not give rise to the rotational-type noise under consideration and need not be considered further in the present analysis. For all other harmonics the expressions in equations (5) and (6) may be written as

$$F_1(t, r, 0) = \frac{1}{\pi} \text{R.P.} \sum_{m=1}^{\infty} \frac{2r}{mbB} \left( \sin \frac{mbB}{2r} \right) e^{imB\Omega \left( t - \frac{b}{2\Omega r} \right)} \frac{dT}{dr} dr d\theta \quad (7)$$

and

$$F_2(t, r, 0) = \frac{1}{\pi} \text{R.P.} \sum_{m=1}^{\infty} \frac{2r}{mbB} \left( \sin \frac{mbB}{2r} \right) e^{imB\Omega \left( t - \frac{b}{2\Omega r} \right)} \frac{1}{r} \frac{dQ}{dr} dr d\theta \quad (8)$$

Although only the real parts of the summations in equations (7) and (8) are necessary in the formulation of the forces, it is convenient to deal with expressions of the forces in complex form throughout the analysis and then, for final results, extract only real parts. Expressions for  $F_1$  and  $F_2$  that pertain to any value of  $\theta$  may be obtained directly from the expressions in equations (7) and (8) for  $\theta = 0$ , because the forces on a second blade element that is shifted with respect to the first by an angle  $\theta$  in the rotational direction are the same as the forces on the first but are retarded by the time  $\theta/\Omega$ . The corresponding Fourier developments are therefore real parts of



$$F_1(t, r, \theta) = \frac{1}{\pi} \sum_{m=1}^{\infty} \frac{2r}{mbB} \left( \sin \frac{mbB}{2r} \right) e^{imB\Omega \left( t - \frac{\theta}{\Omega} - \frac{b}{2\Omega r} \right)} \frac{dT}{dr} dr d\theta \quad (9)$$

and

$$F_2(t, r, \theta) = \frac{1}{\pi} \sum_{m=1}^{\infty} \frac{2r}{mbB} \left( \sin \frac{mbB}{2r} \right) e^{imB\Omega \left( t - \frac{\theta}{\Omega} - \frac{b}{2\Omega r} \right)} \frac{1}{r} \frac{dQ}{dr} dr d\theta \quad (10)$$

It can be seen from this development that, regardless of the form of the chordwise loading employed, that is, as long as the loading remains unchanged throughout the propeller cycle, the variable  $\theta$  will always enter into the Fourier development of the forces in the same manner. As will be discussed subsequently, this relationship leads to a fixed set of integrals in the expressions for the sound field.

With the use of equations (9) and (10) for  $F_1(t, r, \theta)$  and  $F_2(t, r, \theta)$  the equations for the sound pressure associated with thrust and torque (eqs. (1) and (2)) corresponding to rectangular chordwise loading can be written for any harmonic different from zero as

$$p_T = - \frac{e^{i\omega t}}{4\pi^2} \int_0^R \frac{2r}{mbB} \left( \sin \frac{mbB}{2r} \right) \frac{dT}{dr} e^{-\frac{imBb}{2r}} dr \int_0^{2\pi} e^{-imB\theta} \frac{\partial}{\partial x} \frac{e^{-ik\sigma}}{S} d\theta \quad (11a)$$

or

$$p_T = - \frac{e^{i\omega t}}{4\pi^2} \int_0^R \frac{2r}{mbB} \left( \sin \frac{mbB}{2r} \right) \frac{dT}{dr} e^{-\frac{imBb}{2r}} \left[ \frac{ikM_F}{\beta^2} \Pi_1(r) + \frac{ikx}{\beta^2} \Pi_2(r) + x \Pi_3(r) \right] dr \quad (11b)$$

and

$$p_Q = \frac{e^{i\omega t}}{4\pi^2} \int_0^R \frac{2r}{mBb} \left( \sin \frac{mBb}{2r} \right) \frac{1}{r} \frac{dQ}{dr} e^{-\frac{imB}{2r}} dr \int_0^{2\pi} e^{-imB\theta} \frac{1}{r} \frac{\partial}{\partial \theta} \frac{e^{-ik\sigma}}{S} d\theta \quad (12a)$$

or

$$p_Q = \frac{imBe^{i\omega t}}{4\pi^2} \int_0^R \frac{2r}{mBb} \left( \sin \frac{mBb}{2r} \right) \frac{1}{r^2} \frac{dQ}{dr} e^{-\frac{imBb}{2r}} \Pi_1(r) dr \quad (12b)$$

where

$$\Pi_v(r) = \int_0^{2\pi} \frac{e^{-i(mB\theta + k\sigma)}}{s^v} d\theta \quad (v = 1, 2, \text{ and } 3) \quad (13)$$

In the limit  $b = 0$ , these expressions for  $p_T$  and  $p_Q$  are equivalent to those given in equations (21) and (23) of reference 3. This equivalency may be easily verified by use of the limiting value of

$$\lim_{b \rightarrow 0} \left[ \frac{2r}{mBb} \left( \sin \frac{mBb}{2r} \right) e^{-\frac{imBb}{2r}} \right] = 1$$

The integrals in equations (11) to (13) must, in general, be evaluated by approximate or numerical procedures. The integrals  $\Pi_v$  (for  $v = 1, 2, \text{ and } 3$ ) in equation (13) occur in expressions for propeller noise regardless of the form of chordwise loading employed, that is, provided the loading remains unchanged throughout the propeller cycle. These integrals therefore play an important role in the analytical determination of propeller noise and are the integrals for which the treatment and evaluations, pertaining to the near field, constitute the main purpose of this report. In the case of the far field the integrals  $\Pi_v$  can, as was done in references 1 and 3, be reduced to Bessel functions of the first kind. This reduction is, for the sake of completeness, carried out in appendix A in a slightly different and more extended form than was done in references 1 and 3.

The Integrals  $\Pi_v$  in Terms of Lumped Parameters  $mb$ ,  $K_1$ ,

and  $K_2$  and Equivalent Integrals  $I_v$

The integrals  $\Pi_v$  are functions of six parameters ( $x, y, r, M_F, mB$ , and  $k$ ). These parameters can be lumped, however, so that the integrals can be conveniently expressed in terms of related integrals that are functions of only three parameters ( $mB, K_1$ , and  $K_2$ ). This lumping is accomplished as follows:

$$\Pi_v = \int_0^{2\pi} \frac{e^{-i(mB\theta + k\sigma)}}{s^v} d\theta = e^{-\frac{ikx}{\beta^2}} \int_0^{2\pi} \frac{e^{-imB(\theta + \sqrt{x^2 + \beta^2 y^2 + \beta^2 r^2 - 2\beta^2 ry \cos \theta})}}{(x^2 + \beta^2 y^2 + \beta^2 r^2 - 2\beta^2 ry \cos \theta)^{v/2}} d\theta$$

or

$$\Pi_V(r) = \left(\frac{\Omega}{\beta^2 c}\right)^V \frac{e^{-\frac{1kM_R x}{\beta^2}}}{(K_2)^V} I_V(r) = \left(\frac{M_R}{\beta^2 RK_2}\right)^V e^{-\frac{1kM_R x}{\beta^2}} I_V(r) \quad (14)$$

where

$$I_V(r) = \int_0^{2\pi} \frac{e^{-imB(\theta + K_2 \sqrt{1-K_1 \cos \theta})}}{(\sqrt{1-K_1 \cos \theta})^V} d\theta = I_V' - iI_V'' \quad (15)$$

$$K_1 = \frac{2\beta^2 ry}{x^2 + \beta^2 y^2 + \beta^2 r^2} = \frac{2\beta^2 \frac{r}{R} \frac{y}{R}}{\left(\frac{x}{R}\right)^2 + \beta^2 \left(\frac{y}{R}\right)^2 + \beta^2 \left(\frac{r}{R}\right)^2} \quad (16)$$

$$\begin{aligned} K_2 &= \frac{\Omega}{\beta^2 c} \sqrt{x^2 + \beta^2 y^2 + \beta^2 r^2} \\ &= \frac{\Omega R}{\beta^2 c} \sqrt{\left(\frac{x}{R}\right)^2 + \beta^2 \left(\frac{y}{R}\right)^2 + \beta^2 \left(\frac{r}{R}\right)^2} \\ &= \frac{M_R}{\beta^2} \sqrt{\left(\frac{x}{R}\right)^2 + \beta^2 \left(\frac{y}{R}\right)^2 + \beta^2 \left(\frac{r}{R}\right)^2} \end{aligned} \quad (17)$$

and  $M_R$  denotes the Mach number associated with rotational velocity. The integrals  $I_V'$  and  $I_V''$  may be expressed as

$$I_V' = \int_0^{2\pi} \frac{\cos mB(\theta + K_2 \sqrt{1-K_1 \cos \theta})}{(\sqrt{1-K_1 \cos \theta})^V} d\theta \quad (18a)$$

or

$$I_V' = 2e^{-imB\pi} \int_0^\pi \frac{\cos mB\theta \cos mBK_2 \sqrt{1+K_1 \cos \theta}}{(\sqrt{1+K_1 \cos \theta})^V} d\theta \quad (18b)$$

and

$$I_V'' = \int_0^{2\pi} \frac{\sin mB(\theta + K_2 \sqrt{1 - K_1 \cos \theta})}{(\sqrt{1 - K_1 \cos \theta})^v} d\theta \quad (19a)$$

or

$$I_V'' = 2e^{-imB\pi} \int_0^\pi \frac{\cos mB\theta \sin mBK_2 \sqrt{1 + K_1 \cos \theta}}{(\sqrt{1 + K_1 \cos \theta})^v} d\theta \quad (19b)$$

Expressions for the pressures  $p_T$  and  $p_Q$  in terms of the integrals  $I_V$  may be written as

$$p_T = - \frac{M_R^2 e^{imB\left(\Omega t - \frac{M_F M_R}{\beta^2} \frac{x}{R}\right)}}{4\pi^2 \beta^4 R^2 K_2} \int_0^R \frac{2r}{mBb} \left( \sin \frac{mBb}{2r} \right) \frac{dT}{dr} e^{-\frac{imBb}{2r}} \left[ mBM_F I_1'' + \right. \\ \left. \frac{x}{R} \frac{mBM_R}{\beta^2 K_2} I_2'' + \frac{x}{R} \frac{M_R}{\beta^2 K_2^2} I_3' \right] + i \left( mBM_F I_1' + \right. \\ \left. \frac{x}{R} \frac{mBM_R}{\beta^2 K_2} I_2' - \frac{x}{R} \frac{M_R}{\beta^2 K_2^2} I_3'' \right) \Bigg] dr \quad (20)$$

and

$$p_Q = \frac{mBM_R e^{imB\left(\Omega t - \frac{M_F M_R}{\beta^2} \frac{x}{R}\right)}}{4\pi^2 \beta^2 R K_2} \int_0^R \frac{2r}{mBb} \left( \sin \frac{mBb}{2r} \right) \frac{1}{r^2} \frac{dQ}{dr} e^{-\frac{imBb}{2r}} (I_1'' + iI_1') dr \quad (21)$$

#### Tables of $I_V$ Corresponding to the Near Field

Since the integrals  $I_V$  are expressed in terms of lumped parameters  $K_1$  (eq. (16)) and  $K_2$  (eq. (17)), it is necessary to determine the range of values of these parameters that pertain to the near field and to the far field. Regardless of the values that  $x$ ,  $y$ ,  $r$ , and  $\beta$

may have, numerical values of  $K_1$  will always fall in the range

$$0 \leq K_1 \leq 1 \quad (22)$$

but numerical values of  $K_2$  fall in the semi-infinite range

$$0 \leq K_2 \leq \infty \quad (23)$$

Inasmuch as the present development is inherently restricted to propeller tip speeds  $V_t$  that correspond to propeller-tip Mach numbers  $M_t$  less than about unity, the factor  $\frac{M_R}{\beta^2} = \frac{M_R}{1 - M_F^2}$  of  $K_2$  does not become large even when  $M$  approaches 1. This condition exists because

$$M_t = \sqrt{\left(\frac{V_F}{c}\right)^2 + \left(\frac{\Omega R}{c}\right)^2} = \sqrt{M_F^2 + M_R^2} \quad (24)$$

thus, when  $M_F \approx 1$ ,  $M_R \approx 0$  for  $M_t \approx 1$ . It therefore follows that large numerical values of  $K_2$  are associated only with large values of  $\sqrt{x^2 + \beta^2 y^2}$  and these large values imply large distances from the propeller disk. Furthermore, large distances from the propeller disk imply small numerical values of  $K_1$  ( $K_1 \ll 1$ ). Hence, small values of  $K_1$  and large values of  $K_2$  ( $K_2 \gg 1$ ) apply to the far field and large values of  $K_1$  and small values of  $K_2$  pertain to the near field. The specific ranges of values of  $K_1$  and  $K_2$  considered in the present report for the near field are  $0.4 \leq K_1 \leq 0.985$  and  $0.75 \leq K_2 \leq 6.0$ , respectively. These ranges correspond to distances less than about 2.5 propeller diameters from the propeller tip. Tables of values of the integrals  $I_v$  that cover these ranges of  $K_1$  and  $K_2$  for a number of values of the parameter  $mB$  are presented in tables I to VII. Evaluations are presented for values of  $mB$  of 2, 3, 4, 6, 8, 9, and 12; values of  $K_1$  of 0.4, 0.5, 0.6, 0.7, 0.75, 0.8, 0.85, 0.9, 0.925, 0.95, and 0.985; and values of  $K_2$  at intervals of 0.25 in the range  $0.75 \leq K_2 \leq 6.0$ .

The evaluations were made with the use of a card-programmed electronic calculator and are presented, as calculated, to eight significant figures. They can be considered accurate throughout, however, to no more than four significant figures. In order to maintain this accuracy throughout the calculations, it was necessary to decrease the  $\theta$ -increments at which the

integrands were evaluated in making the numerical integrations as the parameter  $mB$  was increased. The specific increments employed are for values of  $mB$  of 2, 3, and 4,  $\Delta\theta = 6^\circ$ ; for a value of  $mB$  of 6,  $\Delta\theta = 4^\circ$ ; and for values of  $mB$  of 8, 9, and 12,  $\Delta\theta = 3^\circ$ . With regard to interpolation in the tables, the increments in  $K_1$  and  $K_2$  are not small enough to yield good results by linear interpolation. Reasonably good interpolated results can be obtained, however, by graphical procedures or by a nonlinear scheme such as Aitken's method.

#### Consideration of Effects of Other Types of Chordwise Loading

It is recalled that the equations for the sound field of propellers derived in foregoing sections of this report are based on the assumption of a uniform chordwise loading. With real propellers, however, the actual form of the chordwise loading, which is, of course, not uniform, may be very significant with regard to the generation of rotational noise. For example, a propeller may be loaded, as in "turn ups", so as to produce no net thrust, yet it may produce considerable noise, mainly of a higher harmonic content. The fact is that negative forces produce noise as well as positive forces and it is only when the separate forces are in one direction that the assumption of uniform loading over the chord should be expected to yield realistic values for the calculated noise. Some examples that bear out this statement and demonstrate the significance of nonuniform chordwise loading are presented in the following paragraphs.

Examples based on square-wave type chordwise loading.— Consider first a propeller to have a constant loading  $F_0$  on the front half-chord and constant loading  $-F_0$  on the rear half-chord of a section of each blade at some radial station  $r$ . That is, let a force acting on each blade be defined by

$$\left. \begin{aligned} f(t, r, \theta) &= \frac{r}{bB} F_0 & (0 < t < \frac{\tau}{2}) \\ f(t, r, \theta) &= -\frac{r}{bB} F_0 & (\frac{\tau}{2} < t < \tau) \\ f(t, r, \theta) &= 0 & (\tau < t < \tau_0) \\ f(t + \tau_0, r, \theta) &= f(t, r, \theta) \end{aligned} \right\} \quad (25)$$

This function corresponds to a zero net force and may be written as a Fourier series, with the zeroth harmonic neglected, as

$$f(t, r, \theta) = \text{R.P.} \frac{4F_0}{\pi} \sum_{l=1}^{\infty} \frac{lr}{mBb} \left( \sin^2 \frac{mB}{4} \frac{b}{r} \right) e^{imB \left( \Omega t - \theta - \frac{b}{2r} \right)} \quad (26)$$

Next, for the sake of comparison, consider the full-wave rectification of  $f(t, r, \theta)$ . That is,

$$\left. \begin{aligned} |f(t, r, \theta)| &= \frac{r}{bB} F_0 & (0 < t < \tau) \\ |f(t, r, \theta)| &= 0 & (\tau < t < \tau_0) \\ |f(t + \tau_0, r, \theta)| &= |f(t, r, \theta)| \end{aligned} \right\} \quad (27)$$

The Fourier expansion of this function is found to be, the zeroth harmonic being neglected

$$|f(t, r, \theta)| = \text{R.P.} \frac{2F_0}{\pi} \sum_{m=1}^{\infty} \frac{r}{mBb} \left( \sin \frac{mBb}{2r} \right) e^{imB \left( \Omega t - \theta - \frac{b}{2r} \right)} \quad (28)$$

It may be noted that there is a phase difference of  $90^\circ$ , indicated by the pure imaginary term in equation (26), between the harmonics of  $f(t, r, \theta)$  and  $|f(t, r, \theta)|$ . The main thing of interest in the present discussion, however, is the relative values of the amplitude functions

$\frac{4r}{mBb} \sin^2 \frac{mBb}{4r}$  of  $f(t, r, \theta)$  and  $\frac{2r}{mBb} \sin \frac{mBb}{2r}$  of  $|f(t, r, \theta)|$ . These

functions are shown plotted as a function of  $\frac{mBb}{2r}$  in figure 2. Exam-

ination of this figure shows that, for a range of values of  $\frac{mBb}{2r} > \frac{\pi}{2}$ , the amplitude function for  $f(t, r, \theta)$  is greater than that for  $|f(t, r, \theta)|$  even though the function  $f(t, r, \theta)$  corresponds to no net force.

Examples based on measured distributions of normal force.- Under actual operating conditions the chordwise distribution of force is usually such that it can be closely approximated with the use of a few simple shapes such as rectangles and triangles. Furthermore, the Fourier development for such shapes can be made once and for all in such a way that, when a chordwise force distribution is known, its Fourier development can be obtained by simple superposition. In order to illustrate this point and to demonstrate further the significance of negative loading with regard to sound pressures, two cases of measured distribution of normal-force coefficients chosen from reference 4 are plotted in figure 3 for the operating conditions shown. Each of the two chosen distributions is for a blade section at  $r = 0.78R$ ; however, one distribution (fig. 3(a)) corresponds to a small net loading with negative forces acting over part of the chord, and the other distribution (fig. 3(b)) corresponds to a large net loading with no negative forces. The area under each curve is divided into rectangles and triangles by the dashed lines shown in figure 3. (The right triangle facing to the right is herein designated right triangle, and the right triangle facing to the left is herein designated left triangle. Their elements are designated right triangular and left triangular, respectively.) The Fourier developments of arbitrarily placed rectangular- and right and left triangular-shaped elements of loading are given in appendix B and have been employed to obtain the Fourier expansions of the distributions in figure 3. The amplitudes so obtained are plotted as a function of  $mB$  in figure 4 and compared with amplitude functions for uniform chordwise loading that yield the same net values as do the distributions in figure 3 and with amplitude functions associated with Dirac delta-type loadings. The results in figure 4(a) are for the case of low net loading and these results show that taking into account the negative forces as well as the positive forces can lead to considerably higher amplitude of sound in the higher harmonics. The results in figure 4(b) for the case of no negative forces show that about the same level of noise is obtained from the uniform distribution of forces as from the detailed distribution of forces. The amplitude functions for the Dirac delta-type loadings agree, as would be expected, with those for rectangular-type loading for lower values of  $mB$ .

#### The Integrations in the Radial Direction

A brief consideration is now given to a method of approximating the integrations with respect to  $r$  indicated in equations (11) and (12). These integrations can be accomplished by a generalization of the concept of effective radius discussed in references 1 to 3. As discussed in these references, the concept of effective radius is based on the consideration that all the sound-producing forces acting on the propeller can be assumed to be concentrated on an annular ring that roughly coincides with the circumference along which the resultant force acts.



In the present case the assumption of an effective radius  $r = R_e$  gives, from equations (20) and (21),

$$\begin{aligned}
 p_T &= - \frac{M_R^2 e^{i m B \left( \Omega t - \frac{M_R M_F}{\beta^2} \frac{x}{R} \right)}}{4 \pi^2 \beta^4 R^2 K_2(R_e)} G_1(R_e) \int_0^R \frac{dT}{dr} dr \\
 &= - \frac{M_R^2 e^{i m B \left( \Omega t - \frac{M_R M_F}{\beta^2} \frac{x}{R} \right)}}{4 \pi^2 \beta^4 R^2 K_2(R_e)} T G_1(R_e)
 \end{aligned} \tag{29}$$

and

$$\begin{aligned}
 p_Q &= \frac{m B M_R e^{i m B \left( \Omega t - \frac{M_F M_R}{\beta^2} \frac{x}{R} \right)}}{4 \pi^2 \beta^2 R K_2(R_e)} G_2(R_e) \int_0^R \frac{dQ}{dr} dr \\
 &= \frac{m B M_R e^{i m B \left( \Omega t - \frac{M_F M_R}{\beta^2} \frac{x}{R} \right)}}{4 \pi^2 \beta^2 R K_2(R_e)} Q G_2(R_e)
 \end{aligned} \tag{30}$$

where

$$\begin{aligned}
 G_1(R_e) &= F_m \left( \frac{m B b}{R_e} \right) \left( \left\{ m B M_F I_1''(R_e) + \frac{x}{R_e} \frac{m B M_R}{\beta^2 K_2(R_e)} I_2''(R_e) + \frac{x}{R} \frac{M_R}{\beta^2 [K_2(R_e)]^2} I_3'(R_e) \right\} + \right. \\
 &\quad \left. i \left\{ m B M_F I_1'(R_e) + \frac{x}{R} \frac{m B M_R}{\beta^2 K_2(R_e)} I_2'(R_e) - \frac{x}{R} \frac{M_R}{\beta^2 [K_2(R_e)]^2} I_3''(R_e) \right\} \right)
 \end{aligned} \tag{31}$$

$$G_2(R_e) = F_m \left( \frac{mBb}{R_e} \right) \frac{1}{R_e^2} \left[ I_1''(R_e) + i I_1'(R_e) \right] \quad (32)$$

and  $F_m \left( \frac{mBb}{R_e} \right)$  is an amplitude function that depends on the form of the chordwise blade loading. For uniform or rectangular-type chordwise loading (see eqs. (20) and (21)),

$$F_m = \frac{2R_e}{mBb} \left( \sin \frac{mBb}{2R_e} \right) e^{-\frac{imBb}{2R_e}}$$

In many cases, especially when the point at which the sound pressure is to be calculated is outside and not too near the cylinder formed by the propeller disk and its wake, the total sound pressure is evidently not sensitive to the details of the radial distribution of forces; thus, the assumption of an effective radius serves as a means of approximating the total sound pressure. See, for example, comparisons of calculated and measured results in references 1 and 2. For cases where details of the radial loading are important, as would be true, for example, at near-field points inside the cylinder formed by the propeller disk and its wake, the effective-radius concept can be generalized as follows:

Suppose the distributions of thrust and torque are graphically represented as indicated in the sketch in figure 5. Divide the areas under the curve into  $n$  parts by dividing the blade length  $R$  into  $n$  parts. The parts into which  $R$  is divided need not be equal but they should be fairly small to insure a reasonably accurate approximation to the integrations. Let  $R_{e,j}$  denote the abscissa of the center of area of the  $j$ th element of area under the curve. Approximate expressions for  $p_T$  and  $p_Q$  (eqs. (20) and (21)) may then be written as

$$p_T = - \frac{M_R^2 e^{imB \left( \Omega t - \frac{M_R M_F}{\beta^2} \frac{x}{R} \right)}}{4\pi^2 \beta^4 R^2} \sum_{j=1}^n \frac{G_1(R_{e,j})}{K_2(R_{e,j})} \Delta T_j \quad (33)$$

and

$$p_Q = \frac{mB M_R e^{imB \left( \Omega t - \frac{M_R M_F}{\beta^2} \frac{x}{R} \right)}}{4\pi^2 \beta^2 R} \sum_{j=1}^n \frac{G_2(R_{e,j})}{K_2(R_{e,j})} \Delta Q_j \quad (34)$$

where  $G_1(R_{e,j})$  and  $G_2(R_{e,j})$  are defined in equations (31) and (32), respectively, and

$$\Delta T_j = \int_{r_j}^{r_{j+1}} \frac{dT}{dr} dr \quad (35)$$

and

$$\Delta Q_j = \int_{r_j}^{r_{j+1}} \frac{dQ}{dr} dr \quad (36)$$

are the  $j$ th elements of area under the radial distributions of thrust and torque curves.

#### Root-Mean-Square Value of Pressure

Before some specific examples are discussed, it is appropriate to point out that, in applications of propeller noise theory, one is generally interested in the magnitude of the root mean square of the total pressure  $p_{rms}$  for each harmonic of noise. In order to obtain this quantity, it is convenient to treat the propeller forces as complex quantities. As may be noted in equations (20) and (21) or equations (33) and (34) the pressures  $p_T$  and  $p_Q$  are of forms

$$p_T = (\cos \omega t + i \sin \omega t)(A_1 + iA_2) \quad (37)$$

and

$$p_Q = (\cos \omega t + i \sin \omega t)(B_1 + iB_2) \quad (38)$$

The form of an expression for the total sound pressure  $p$  associated with thrust and torque is obtained by adding these expressions and retaining only the real parts of the sum. That is,

$$p = R.P.(p_T + p_Q) = (A_1 + B_1)\cos \omega t - (A_2 + B_2)\sin \omega t \quad (39)$$

The root-mean-square value of this expression is

$$p_{rms} = \left\{ \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \left[ (A_1 + B_1) \cos \omega t - (A_2 + B_2) \sin \omega t \right]^2 dt \right\}^{1/2}$$

$$= \frac{1}{\sqrt{2}} \sqrt{(A_1 + B_1)^2 + (A_2 + B_2)^2} = \frac{1}{\sqrt{2}} |p_T + p_Q| \quad (40)$$

Thus  $p_{rms}$  is obtained by simply multiplying the absolute value of the complex sum  $p_T + p_Q$  by  $\frac{1}{\sqrt{2}}$ .

### SOME APPLICATIONS TO SPECIFIC PROPELLERS

The propellers chosen for the applications of the tables are three- and four-blade 16-foot-diameter propellers with solidity factors ranging from 0.12 to 0.16. The efficiency, total thrust, and total torque for such propellers have been determined from charts of references 5 and 6 and are summarized in figure 6. The total thrust for three different altitudes and the efficiency are presented as functions of flight Mach number  $M_F$  in figures 6(a) and 6(b), respectively. The total torque is presented as a function of rotational Mach number  $M_R$  in figure 6(c).

The present examples are based on the assumption of uniform chord-wise loading and, except for one case, on the concept of an effective radius (eqs. (29) and (30)) with  $R_e = 0.8R$ . When the calculated results were obtained, the necessary interpolations in the tables of  $I_v$  were performed by graphical procedures.

The first results are presented in figure 7 and are intended to give some indication as to the differences that might be expected between calculations based on the concept of an effective radius and calculations based on radial distributions of forces. Shown in the figure are values of the root-mean-square pressure, associated with the first harmonic of a four-blade 16-foot-diameter propeller, calculated with the use of equations (33) and (34) (by using five steps), for radial distributions of forces shown in the upper plot and calculated with the use of equations (29) and (30) for the forces assumed to be concentrated on an annulus of radius  $0.8R$ . Calculations by each procedure were made for operating conditions indicated in figure 7 at various values of  $x$  in the range  $-0.3D \leq x \leq 0.3D$  along a line  $y = 0.7D$  (11.2 feet); that is, along a line two-fifths of the propeller radius from the tip and extending a distance three-fifths of the radius behind to three-fifths of the radius ahead of the plane of rotation. For the conditions considered, the pressure or sound levels based on distributed forces are below those

based on effective radius or concentrated forces at each point considered. The significant point is that the shapes of the two curves, considered as functions of  $x/D$ , are about the same; this fact indicates that calculations based on an effective radius can at least be used for studying trends.

In figure 8 calculated values of the root-mean-square pressure associated with the first harmonic of a four-blade 16-foot-diameter propeller are plotted as functions of  $x/D$  for two altitude conditions, three values of rotational Mach number  $M_R$ , and a fixed value (0.6) of flight Mach number  $M_F$ .

As can be noted in figure 8 for the set of conditions under consideration, the peaks or highest amplitudes of pressure occur behind the plane of rotation at values of  $x/D$  in the range from -0.05 to -0.125. A somewhat surprising feature of the results is the rather sharp rearward trend of the position of peak amplitude as  $M_R$  is increased. As noted in reference 3, the trend of position of peak amplitude of pressure with increasing  $M_F$  is from some rearward position toward the plane of rotation or just the opposite of that indicated in figure 7 for increasing  $M_R$ . The results also indicate a rather sharp attenuation of sound-pressure level with altitude and with distance away from the location of peak amplitude. The apparent attenuation with altitude, however, can be directly associated with the decrease in thrust and torque (see fig. 6) required for the higher altitude.

In figure 9 the calculated root-mean-square pressures associated with the first harmonic of a four-blade, 16-foot-diameter propeller for a point  $x = -0.15D$ ,  $y = 11.2$  feet (or  $0.7D$ ) are shown plotted as a function of flight Mach number for three altitude conditions and a fixed rotational Mach number ( $M_R = 0.8$ ). These results show, as do the examples in reference 3, that, for a fixed altitude, the amplitudes of sound pressure decrease substantially with flight Mach number for Mach numbers up to about 0.4 or 0.5 and then increase rapidly with flight Mach number.

Figure 10 shows the effect of flight Mach number on the calculated root-mean-square pressures for various values of the parameter  $mB$  for two altitude conditions. The calculations are for a point  $x = 0$ ,  $y = 11.2$  feet (or  $0.7D$ ) and for a fixed value of the rotational Mach number ( $M_R = 0.7$ ). A change in the parameter  $mB$  may correspond to either a change in number of propeller blades  $B$  or a change in harmonic  $m$ . For example, the curves in figure 10 for  $mB = 8$  may be interpreted as the calculated sound pressure corresponding to either the first harmonic of an 8-blade propeller, the second harmonic of a 4-blade propeller, or the fourth harmonic of a 2-blade propeller.

All the curves in figure 10 show the falling-off or diminishing characteristic as the Mach number  $M_F$  is increased from zero up to 0.2 or 0.3 and then the rapid increase with Mach number when  $M_F$  exceeds about 0.4 or 0.5. The total amount that the calculated pressures decrease from the  $M_F = 0$  condition is about the same percentage (about 20) in each case. It may be implied therefore that, in the neighborhood of take-off conditions, the Mach number effect is about the same, percentagewise, regardless of the value of  $mB$ . The increase with Mach number, after a Mach number of 0.4 or 0.5 has been attained, however, depends strongly on the value of  $mB$  and becomes very large as  $mB$  increases beyond 6 or 8. For these large values of  $mB$  the present simplified theory, especially with the assumption of an effective radius, could only serve to indicate the trends roughly. In order to yield realistic magnitudes, it is likely that the details of the radial loading and, perhaps, details of the chordwise loadings would have to be taken into account.

The effect of distance from the propeller on the sound level along a line of constant  $y$  is implied in figures 7 and 8. Figure 11 shows the corresponding effect along a line of constant  $x$  ( $x = -0.15D$ ). The results shown are the calculated root-mean-square pressures corresponding to the first harmonic of a four-blade 16-foot-diameter propeller for  $M_F = 0$  and  $M_R = 0.7$  at sea level plotted as a function of  $d/D$  where  $d$  is the distance from the propeller tip. Figure 11 shows that, for the conditions considered, the amplitude of pressure drops off very rapidly as the distance from the propeller tip is increased. This type of dropoff is typical of that expected for other values of the parameters  $M_F$ ,  $M_R$ , and  $mB$ .

#### CONCLUDING REMARKS

A main purpose of this report is to present tabulated values of certain definite integrals that are involved in the calculation of near-field propeller noise. These tabulations cover a wide range of operating conditions and are useful for estimating lower harmonic, near-field propeller noise when either the concept of an effective radius or radial distributions of forces are considered.

Consideration is given to the general forms of chordwise and radial load distributions and it is shown that propellers operating even with zero net forces can give rise to considerable noise, especially in the

higher harmonics. Results of applications of noise theory to some specific 16-foot-diameter propellers are presented and discussed.

Langley Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Langley Field, Va., June 26, 1956.

## APPENDIX A

EVALUATION OF THE INTEGRALS  $I_\nu$  FOR THE FAR FIELD

As pointed out in the text, the integrals  $I_\nu$  corresponding to the far field or small values of the parameter  $K_1$  ( $K_1 \ll 1$ ) can be expressed in terms of Bessel functions of the first kind. The purpose of this appendix is to reduce the integrals to such expressions.

For this purpose it is convenient to consider the integral expression for  $I_\nu$  given in equation (15); that is,

$$I_\nu(r) = \int_0^{2\pi} \frac{e^{-imB(\theta + K_2 \sqrt{1 - K_1 \cos \theta})}}{(\sqrt{1 - K_1 \cos \theta})^\nu} d\theta = I_\nu' - iI_\nu''$$

The value of the radical  $\sqrt{1 - K_1 \cos \theta}$  appearing in the exponential term and in the denominator of the integrand of this expression is, for  $K_1 \ll 1$ , closely approximated by

$$\sqrt{1 - K_1 \cos \theta} \approx 1 - \frac{K_1}{2} \cos \theta \quad (A1)$$

Correspondingly, the integral  $I_\nu$  is closely approximated by

$$I_\nu \approx \int_0^{2\pi} \frac{e^{-imB\left(\theta + K_2 - \frac{K_1 K_2}{2} \cos \theta\right)}}{\left(1 - \frac{K_1}{2} \cos \theta\right)^\nu} d\theta$$

or

$$I_\nu \approx e^{-imBK_2} \int_0^{2\pi} e^{-imB\left(\theta - \frac{K_1 K_2}{2} \cos \theta\right)} \left(1 + \frac{\nu K_1}{2} \cos \theta\right) d\theta \quad (A2)$$



From the known integral representation of Bessel functions,

$$\int_0^{2\pi} e^{-i(n\theta - \lambda \cos \theta)} d\theta = 2\pi i^n J_n(\lambda) \quad (A3)$$

and

$$\begin{aligned} \cos \theta e^{-i(n\theta - \lambda \cos \theta)} d\theta &= -i \frac{\partial}{\partial \lambda} \int_0^{2\pi} e^{-i(n\theta - \lambda \cos \theta)} d\theta \\ &= -\pi i^{n+1} [J_{n-1}(\lambda) - J_{n+1}(\lambda)] \end{aligned} \quad (A4)$$

The expression for  $I_v$  given in equation (A2) may be written as

$$\begin{aligned} I_v &\approx \pi i^{mB} e^{-imBK_2} \left\{ 2J_{mB} \left( \frac{mBK_1 K_2}{2} \right) - \right. \\ &\quad \left. \frac{ivK_1}{2} \left[ J_{mB-1} \left( \frac{mBK_1 K_2}{2} \right) - J_{mB+1} \left( \frac{mBK_1 K_2}{2} \right) \right] \right\} \end{aligned} \quad (A5)$$

or, by substituting the relations for  $K_1$  and  $K_2$ ,

$$\begin{aligned} I_v &\approx \pi i^{mB} e^{-\frac{ik}{\beta^2} \sqrt{x^2 + \beta^2 y^2 + \beta^2 r^2}} \left\{ 2J_{mB} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2 + \beta^2 r^2}} \right) - \right. \\ &\quad \left. i \frac{v\beta^2 ry}{x^2 + \beta^2 y^2 + \beta^2 r^2} \left[ J_{mB-1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2 + \beta^2 r^2}} \right) - J_{mB+1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2 + \beta^2 r^2}} \right) \right] \right\} \end{aligned} \quad (A6)$$

or, since  $\beta r \ll \sqrt{x^2 + \beta^2 y^2}$  for large distances from the propeller disk,

$$\begin{aligned}
I_v &\approx \pi i^{mB} e^{-\frac{ik}{\beta^2} \sqrt{x^2 + \beta^2 y^2}} \left\{ 2J_{mB} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) - \right. \\
&\quad \left. \frac{iv\beta^2 ry}{x^2 + \beta^2 y^2} \left[ J_{mB-1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) - J_{mB+1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) \right] \right\} \\
&= \pi i^{mB} \left( 2 \cos \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} J_{mB} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) - \right. \\
&\quad \frac{v\beta^2 ry}{x^2 + \beta^2 y^2} \left( \sin \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} \right) \left[ J_{mB-1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) - J_{mB+1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) \right] - \\
&\quad i \left\{ \left( 2 \sin \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} \right) J_{mB} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) + \right. \\
&\quad \left. \frac{v\beta^2 ry}{x^2 + \beta^2 y^2} \left( \cos \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} \right) \left[ J_{mB-1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) - J_{mB+1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) \right] \right\} \right) \\
&\hspace{25em} (A7)
\end{aligned}$$

Thus, for  $mB$  even,

$$\begin{aligned}
I_v' &= \pi (-1)^{mB/2} \left\{ \left( 2 \cos \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} \right) J_{mB} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) - \right. \\
&\quad \left. \frac{v\beta^2 ry}{x^2 + \beta^2 y^2} \left( \sin \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} \right) \left[ J_{mB-1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) - J_{mB+1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) \right] \right\} \\
&\hspace{25em} (A8)
\end{aligned}$$

and

$$\begin{aligned}
I_v'' &= \pi (-1)^{mB/2} \left\{ \left( 2 \sin \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} \right) J_{mB} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) + \right. \\
&\quad \left. \frac{v\beta^2 ry}{x^2 + \beta^2 y^2} \left( \cos \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} \right) \left[ J_{mB-1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) - J_{mB+1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) \right] \right\} \\
&\hspace{25em} (A9)
\end{aligned}$$

For  $mB$  odd,

$$I_v' = \pi(-1)^{\frac{mB-1}{2}} \left\{ \left( 2 \sin \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} \right) J_{mB} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) + \right. \\ \left. \frac{v\beta^2 ry}{x^2 + \beta^2 y^2} \left( \cos \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} \right) \left[ J_{mB-1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) - J_{mB+1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) \right] \right\} \quad (A10)$$

and

$$I_v'' = \pi(-1)^{\frac{mB+1}{2}} \left\{ \left( 2 \cos \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} \right) J_{mB} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) - \right. \\ \left. \frac{v\beta^2 ry}{x^2 + \beta^2 y^2} \left( \sin \frac{k}{\beta^2} \sqrt{x^2 + \beta^2 y^2} \right) \left[ J_{mB-1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) - J_{mB+1} \left( \frac{kry}{\sqrt{x^2 + \beta^2 y^2}} \right) \right] \right\} \quad (A11)$$

These approximations of  $I_v$  for the far field go a step beyond that given in reference 3 in that the terms herein involving  $v$  as a factor were not considered in reference 3. It is not deemed necessary to tabulate  $I_v$  for the far-field case since tabulations of the Bessel functions  $J_n$  are so abundant in the literature.

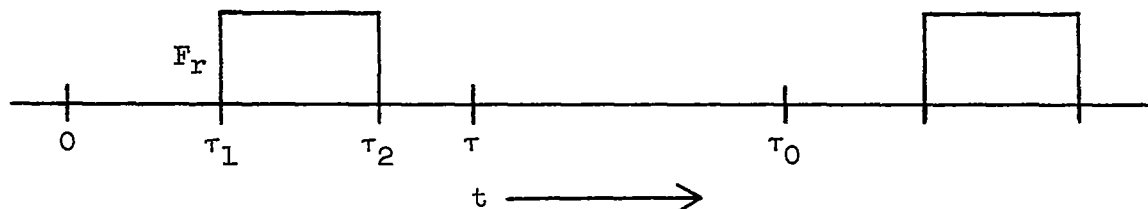
## APPENDIX B

THE FOURIER DEVELOPMENT OF SOME SIMPLE SHAPES USEFUL FOR  
REPRESENTING CHORDWISE FORCE DISTRIBUTIONS  
FOR PROPELLER BLADES

It is assumed that any chordwise distribution of force acting on a propeller-blade section can be satisfactorily represented by superimposing rectangular- and triangular-shaped elements of force as indicated in figure 3. This, at least, is a convenient assumption because such elements can be expanded into Fourier series in such a way as to make the Fourier development of the force distribution obtainable by simply adding the Fourier series representations of the different components into which the force distribution is divided.

Rectangular Element of Loading

Consider first a rectangular element of periodic force  $f_r(t, r, \theta)$  that is located on some element of chord in a force-time system.



If the origin of coordinates is chosen to be the leading edge of the blade section,  $f_r$  may be expressed as follows:

$$\left. \begin{aligned} f_r(t, r, \theta) &= 0 & 0 < t < \tau_1 \\ f_r(t, r, \theta) &= F_r \, d\theta \, dr & \tau_1 < t < \tau_2 \\ f_r(t, r, \theta) &= 0 & \tau_2 < t < \tau_0 \\ f_r(t + \tau_0, r, \theta) &= f_r(t, r, \theta) \end{aligned} \right\} \quad (B1)$$

where

$$\tau_1 = \mu_1 \tau = \mu_1 \frac{b}{r\Omega}$$

$$\tau_2 = \mu_2 \tau = \mu_2 \frac{b}{r\Omega}$$

The terms  $\mu_1$  and  $\mu_2$  are constants such that  $0 \leq \mu_1 \leq 1$  and  $0 \leq \mu_2 \leq 1$ . Equations (B1) may be expressed as a Fourier series, the term corresponding to the zeroeth harmonic being neglected, as

$$\begin{aligned} f_r(t, r, \theta) &= \frac{2F_r}{\tau_0} \frac{d\theta}{dr} \sum_1^{\infty} \left[ \cos \frac{2m\pi}{\tau_0} \left( t - \frac{\theta}{\Omega} \right) \int_{\tau_1}^{\tau_2} \cos \frac{2m\pi}{\tau_0} t' dt' + \right. \\ &\quad \left. \sin \frac{2m\pi}{\tau_0} \left( t - \frac{\theta}{\Omega} \right) \int_{\tau_1}^{\tau_2} \sin \frac{2m\pi}{\tau_0} t' dt' \right] \\ &= \frac{2F_r}{\pi} \frac{d\theta}{dr} \sum_1^{\infty} \frac{1}{m} \left[ \cos \frac{2m\pi}{\tau_0} \left( t - \frac{\theta}{\Omega} \right) \cos \frac{m\pi}{\tau_0} (\tau_2 + \tau_1) \sin \frac{m\pi}{\tau_0} (\tau_2 - \tau_1) + \right. \\ &\quad \left. \sin \frac{2m\pi}{\tau_0} \left( t - \frac{\theta}{\Omega} \right) \sin \frac{m\pi}{\tau_0} (\tau_2 + \tau_1) \sin \frac{m\pi}{\tau_0} (\tau_2 - \tau_1) \right] \\ &= \frac{2F_r}{\pi} \frac{d\theta}{dr} \sum_1^{\infty} \frac{1}{m} \left[ \cos \frac{2m\pi}{\tau_0} \left( t - \frac{\theta}{\Omega} - \frac{\tau_1 + \tau_2}{2} \right) \sin \frac{m\pi}{\tau_0} (\tau_2 - \tau_1) \right] \end{aligned} \quad (B2)$$

This expression may be written in complex form as

$$f_r(t, r, \theta) = \text{R.P.} \frac{2F_r}{\pi} \sum_1^{\infty} \frac{1}{m} e^{\frac{2im\pi}{\tau_0} \left( t - \frac{\theta}{\Omega} \right)} \left[ \sin \frac{m\pi}{\tau_0} (\tau_2 - \tau_1) \right] e^{-\frac{im\pi}{\tau_0} (\tau_1 + \tau_2)} d\theta dr \quad (B3)$$

or

$$f_r(t, r, \theta) = \text{R.P.} \frac{2F_r}{\pi} \sum_1^{\infty} \frac{e^{imB(\Omega t - \theta)}}{m} \left[ \sin \frac{mBb}{2r} (\mu_2 - \mu_1) \right] e^{-\frac{imBb}{2r} (\mu_2 + \mu_1)} d\theta dr \quad (B4)$$

The total force represented by  $f_r(t, r, \theta)$  that acts on a blade section may be seen from equation (B1) to be

$$f_r(t, r, \theta) = F_r(\mu_2 - \mu_1) \tau \, d\theta \, dr \quad (B5)$$

If this force is to represent some preassigned percentage  $h_r$  of a total force of magnitude  $F$ , such as total thrust or total torque, that acts on the section, then

$$F_r(\mu_2 - \mu_1) \tau = \frac{h_r r}{bB} F \tau \quad (B6)$$

or

$$F_r = \frac{h_r}{\mu_2 - \mu_1} \frac{r}{Bb} F \quad (B7)$$

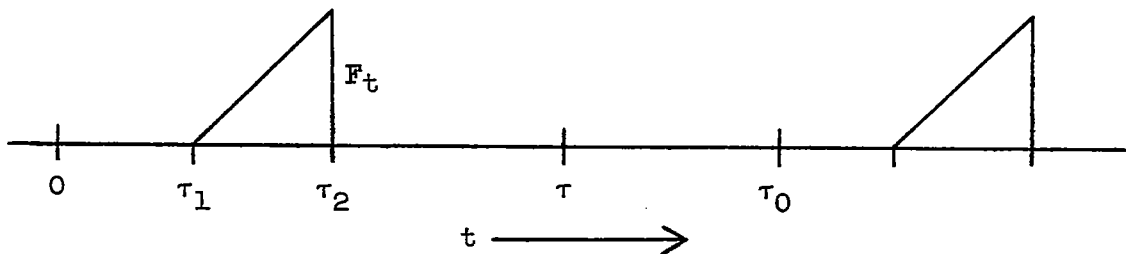
Substituting this expression for  $F_r$  into equation (B4) gives

$$f_r(t, r, \theta) = R.P. \frac{h_r F}{\pi(\mu_2 - \mu_1)} \sum_1^{\infty} \frac{2r}{mBb} e^{imB(\Omega t - \theta)} \left[ \sin \frac{mBb}{2r} (\mu_2 - \mu_1) \right] e^{-\frac{imBb}{2r} (\mu_2 + \mu_1)} \, d\theta \, dr \quad (B8)$$

Note that in this equation, if  $\mu_1 = 0$ ,  $\mu_2 = 1$ , and  $h_r F = \frac{dT}{dr}$ , the results given in equation (9) are obtained. If  $\mu_1 = 0$ ,  $\mu_2 = 1$ , and  $h_r F = \frac{1}{r} \frac{dQ}{dr}$ , the results given in equation (10) are obtained.

#### Left-Triangular Element of Loading

Consider next a left-triangular element of loading  $f_{lt}(t, r, \theta)$  on the chord element  $\tau_1 \tau_2$  in a time-coordinate system (see sketch).



The equation for this case may be expressed as

$$\left. \begin{aligned} f_{lt}(t, r, \theta) &= 0 & (0 < t < \tau_1) \\ f_{lt}(t, r, \theta) &= \frac{F_t(t - \tau_1)}{\tau_2 - \tau_1} d\theta dr & (\tau_1 < t < \tau_2) \\ f_{lt}(t, r, \theta) &= 0 & (\tau_2 < t < \tau_0) \\ f_{lt}(t + \tau_0, r, \theta) &= f_{lt}(t, r, \theta) \end{aligned} \right\} \quad (B9)$$

The Fourier expansion for this function is

$$\begin{aligned} f_{lt}(t, r, \theta) &= \frac{2F_t}{\tau_0(\tau_2 - \tau_1)} \sum_1^{\infty} \left[ \cos \frac{2m\pi}{\tau_0} \left( t - \frac{\theta}{\Omega} \right) \int_{\tau_1}^{\tau_2} (t' - \tau_1) \cos \frac{2m\pi}{\tau_0} t' dt' + \right. \\ &\quad \left. \sin \frac{2m\pi}{\tau_0} \left( t - \frac{\theta}{\Omega} \right) \int_{\tau_1}^{\tau_2} (t' - \tau_1) \sin \frac{2m\pi}{\tau_0} t' dt' \right] \\ &= \frac{F_t}{\pi(\tau_2 - \tau_1)} \sum_1^{\infty} \left[ -\frac{\tau_2 - \tau_1}{m} \sin \frac{2m\pi}{\tau_0} \left( t - \frac{\theta}{\Omega} - \tau_2 \right) + \right. \\ &\quad \left. \frac{\tau_0}{m^2\pi} \sin \frac{m\pi}{\tau_0} (\tau_2 - \tau_1) \sin \frac{2m\pi}{\tau_0} \left( t - \frac{\theta}{\Omega} - \frac{\tau_1 + \tau_2}{2} \right) \right] \end{aligned} \quad (B10)$$

or in complex form

$$\begin{aligned} f_{lt}(t, r, \theta) &= \text{R.P.} \frac{F_t}{\pi(\tau_2 - \tau_1)} \sum_1^{\infty} i e^{\frac{2im\pi}{\tau_0} \left( t - \frac{\theta}{\Omega} \right)} \left\{ \frac{\tau_2 - \tau_1}{m} e^{-\frac{2im\pi\tau_2}{\tau_0}} - \right. \\ &\quad \left. \frac{\tau_0}{\pi m^2} \left[ \sin \frac{m\pi}{\tau_0} (\tau_2 - \tau_1) \right] e^{-\frac{im\pi}{\tau_0} (\tau_1 + \tau_2)} \right\} \end{aligned} \quad (B11)$$

or

$$\begin{aligned} f_{lt}(t, r, \theta) &= \text{R.P.} \frac{F_t}{\pi} \sum_1^{\infty} i \frac{e^{imB(\Omega t - \theta)}}{m} \left\{ e^{-\frac{imBb}{r} \mu_2} - \right. \\ &\quad \left. \frac{2r}{mBb(\mu_2 - \mu_1)} \left[ \sin \frac{mBb}{2r} (\mu_2 - \mu_1) \right] e^{-\frac{imBb}{2r} (\mu_2 + \mu_1)} \right\} \end{aligned} \quad (B12)$$

As in the preceding paragraph, if the total force acting on a blade section is to represent some preassigned percentage  $h_t$  of a total force  $F$ , then

$$F_t = \frac{2h_t r}{(\mu_2 - \mu_1)} \frac{r}{Bb} F \quad (B13)$$

and

$$f_{lt}(t, r, \theta) = \text{R.P.} \frac{h_{lt} F}{\pi(\mu_2 - \mu_1)} \sum_1^{\infty} \frac{2i r e^{imB(\Omega t - \theta)}}{mBb} \left\{ e^{-\frac{imBb}{r} \mu_2} - \frac{2r}{mBb(\mu_2 - \mu_1)} \left[ \sin \frac{mBb}{2r} (\mu_2 - \mu_1) \right] e^{-\frac{imBb}{2r} (\mu_2 + \mu_1)} \right\} dr d\theta \quad (B14)$$

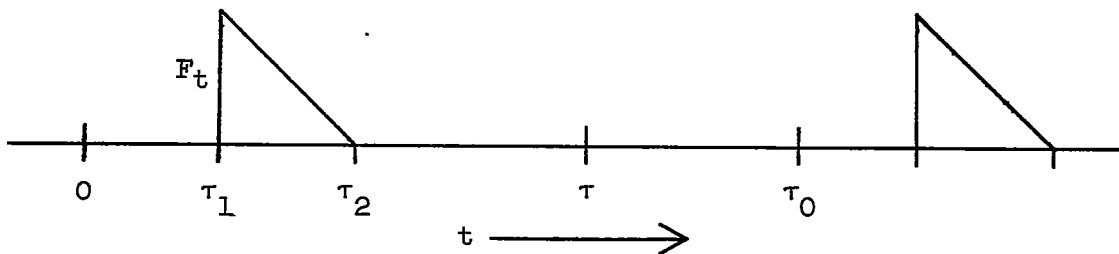
This result is easily checked by considering the limit

$$\lim_{\mu_1 \rightarrow \mu_2} f_{lt}(t, r, \theta) \quad (B15)$$

This limit leads, as it should, to the Fourier expansion of the Dirac delta function of strength  $h_{lt} F$ .

#### Right-Triangular Element of Loading

A right-triangular element of loading  $f_{rt}$  in a time-coordinate system is shown in the following sketch:





The equation for this element of loading may be written as

$$\left. \begin{aligned} f_{rt}(t, r, \theta) &= 0 & (0 < t < \tau_1) \\ f_{rt}(t, r, \theta) &= -\frac{F_t(t - \tau_2)}{\tau_2 - \tau_1} & (\tau_1 < t < \tau_2) \\ f_{rt}(t, r, \theta) &= 0 & (\tau_2 < t < \tau) \\ f_{rt}(t + \tau_0, r, \theta) &= f_{rt}(t, r, \theta) \end{aligned} \right\} \quad (B16)$$

The Fourier expansion of this equation can be obtained from equation (B14) by simply interchanging  $\mu_1$  and  $\mu_2$ . That is,

$$f_{rt}(t, r, \theta) = -R.P. \frac{h_{rt} F}{\pi(\mu_2 - \mu_1)} \sum_1^{\infty} \frac{2i r e^{imB(\Omega t - \theta)}}{mBb} \left\{ e^{-\frac{imBb}{r} \mu_1} - \frac{2r}{mBb(\mu_2 - \mu_1)} \left[ \sin \frac{mBb}{2r} (\mu_2 - \mu_1) \right] e^{-\frac{imBb}{2r} (\mu_2 + \mu_1)} \right\} dr d\theta \quad (B17)$$

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TABLE I.- VALUES OF THE INTEGRAL  $I_1$  FOR  $m = 2$ (a)  $I_1'$ 

$k_2 \backslash k_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.15740477	0.26185549	0.41100003	0.63076358	0.78344608	0.98262824	1.2573318	1.6743358	1.9894793	2.4560452	3.9553356
1.0	.20129075	.33250404	.51680176	.78228643	.96305196	1.1914901	1.5075615	1.9707001	2.3132179	2.8114629	4.3681990
1.25	.24305216	.40109444	.62201442	.93744078	1.1497220	1.418445	1.7768853	2.2963038	2.6723681	3.2102675	4.8400044
1.5	.29229673	.48048921	.7408921	1.0081107	1.2440054	1.5438008	1.9420805	2.515184	2.9260391	3.5059026	5.2166602
1.75	.39050743	.63229132	.94833411	.8876032	1.1195240	1.4280419	1.8462901	2.4564380	2.8946821	3.5115576	5.3017234
2.0	.02707379	.08785947	.21724289	.46963152	.67194751	.95400731	1.3592005	1.9756860	2.8289158	3.0718939	4.9322010
2.25	-.24232753	-.32056999	-.34646473	-.25273869	-.12481917	.0745277	.44438211	1.0325293	1.8846012	2.1406715	4.0595045
2.5	-.51073001	-.83992925	-1.0733165	-1.2001658	-1.1802554	-1.0644984	-.79741107	-.26942432	-1.6882171	-.82676565	2.7957927
2.75	-.90958072	-1.3536152	-1.8085738	-2.1821082	-2.2880475	-2.2931330	-2.1336769	-1.6862642	-1.2718714	-.61876637	1.3976334
3.0	-1.1199484	-1.7037759	-2.3435130	-2.9434572	-3.1734787	-3.3031964	-3.2602364	-2.9061465	-2.5205096	-1.8773935	1.18566204
3.25	-1.1059263	-1.7355177	-2.4748596	-3.2399638	-3.5757666	-3.8223117	-3.8982677	-3.6179637	-3.297286	-2.6744858	-.57126580
3.5	-.80051938	-1.3502080	-2.0746911	-2.9202607	-3.6913421	-4.2928083					
3.75	-.20921306	-.55467668	-1.1466059	-1.9901144	-.62661362						
4.0	.57439147	.52410994	.15420710	-.62661362							
4.25	1.3784726	1.6589062	1.5482380	.86342967							
4.5	1.994746	2.5601051	2.7032202								
4.75	2.2138202	2.9719568	3.276358								
5.0	1.9289725	2.7407564									
5.25	1.1373823	1.8710056									
5.5	-.01932900	.53690696									
5.75	-1.2871545	-.95808134									
6.0	-2.3570058	-2.2560971									

(b)  $I_1''$ 

$k_2 \backslash k_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.01350690	0.02110769	0.03040069	0.04138838	0.04752276	0.05407286	0.06105663	0.06845622	0.0722485	0.07629471	0.08203170
1.0	.04972472	.07844825	.11225662	.15117005	.17573143	.20025746	.22484466	.25370450	.28507434	.32822718	.38419755
1.25	.12797738	.20024253	.28883901	.3992948	.45272031	.51570777	.58292607	.65440141	.69174717	.73016885	.78577899
1.5	.25474304	.3992948	.57720113	.78920538	.90826642	1.0361911	1.1731125	1.3191497	1.3956741	1.4745185	1.5888731
1.75	.41572259	.65386865	.94915544	1.3041134	1.5049356	1.7218360	1.9552891	2.2057833	2.3375816	2.4738367	2.6722171
2.0	.56893093	.90081006	1.3181049	1.8279085	2.1200941	2.4386913	2.7849858	3.1679416	3.5993814	3.5668409	3.8693675
2.25	.69046586	1.0413549	1.5510660	2.1899650	2.5647358	2.9800922	3.4390950	3.9440398	4.2167112	4.5014643	4.9230152
2.5	.97078093	.97799416	1.5058925	2.2083945	2.6379636	3.1272197	3.6826887	4.1941407	4.6554070	5.0206448	5.5694906
2.75	.34386349	.63344010	1.0614049	1.7574413	2.2006613	2.7296874	3.3560219	4.0930635	4.5075537	4.9552227	5.6415287
3.0	-.08958536	-.00867592	.28091710	.82738301	1.2419140	1.7748678	2.4469474	3.2814113	3.7668708	4.3028626	5.1445637
3.25	-.64512099	-.80549060	-.78440147	-.44594622	-.09433560	.11533810	1.1160417	2.0450253	2.6080298	3.2442215	4.2694330
3.5	-1.1983192	-1.6311761	-1.9002135	-1.6070161		-1.0621308					
3.75	-1.5904496	-2.2508842	-2.7745629	-2.9174240		-2.3189916					
4.0	-1.6716927	-2.4504125	-3.1542736	-3.5111906							
4.25	-1.3504681	-2.1049530	-2.6878947	-3.4256982							
4.5	-.63320966	-1.2211179	-1.9851593								
4.75	.35996632	.04503650									
5.0	1.4123442	1.4161924									
5.25	2.2595576	2.5617211									
5.5	2.9490734	3.1871242									
5.75	2.4607534	3.1188119									
6.0	1.6589401	2.3573958									

TABLE I.- VALUES OF THE INTEGRAL  $I_1$  FOR  $m = 2$  - Continued

(a)  $I_2'$

$K_1$ $K_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.29521042	0.51539787	0.86174044	1.4564637	1.9233477	2.6038468	3.6809421	5.6120098	7.4109188	10.525224	25.538569
1.0	.28066053	.49281379	.83220931	1.4120775	1.8724745	2.5429494	3.6155577	5.5686843	7.3334459	10.443491	25.450876
1.25	.23819112	.42638991	.73644402	1.2816084	1.7225193	2.3752446	3.4226154	5.3553042	7.1046250	10.202012	25.190891
1.5	.14444495	.27957661	.52444358	.99210416	1.3895692	1.9956853	2.9932416	4.4687196	6.5944486	9.6632493	24.610714
1.75	-.02241464	.01759736	.14497046	.47201547	.79023455	1.3109753	2.2169014	3.9965040	5.6686080	8.6842970	23.554572
2.0	-.27011597	-.37322017	-.42448610	-.31365831	-.11884207	.06816843	1.0295650	2.6521476	4.2430319	7.1733719	21.918991
2.25	-.57937951	-.86685805	-.1.15065315	-.1.3290160	-.1.3015202	-.1.0985595	-.53889950	.86639449	2.3365687	5.1441900	19.709073
2.5	-.8972634	-1.3817469	-1.9289534	-2.4462791	-2.6213255	-2.4458234	-2.340628	-1.2226836	.09658740	2.7417657	17.064390
2.75	-1.1386299	-1.7969168	-2.5931173	-3.4585922	-3.8536950	-4.1342040	-4.1256926	-3.3465956	-2.7201409	.22194590	14.236326
3.0	-1.8091374	-1.9679575	-2.91488017	-4.1227686	-4.7343089	-5.2816722	-5.5986323	-5.2157480	-4.3111069	-2.1115646	11.518777
3.25	-1.0263461	-1.7735279	-2.4290470	-4.220267	-5.0319608	-5.8408353	-6.5015494	-6.5562645	-5.9165575	-4.0124559	9.1555095
3.5	-.56420387	-1.1592506	-2.1529874	-3.6631154	-4.669827	-5.6768152	-6.8159139				
3.75	.14308239	-.17551489	-.96843957	-2.4669827	-3.3353444	-4.3191222					
4.0	.97409963	1.0207426	.53845131	.92995030							
4.25	1.7471389	2.1833015	2.7772258								
4.5	2.2579079	3.0351027	3.3191222								
4.75	2.3337949	3.3399963	3.9669235								
5.0	1.8877974	2.9722565									
5.25	.95576281	1.9616090									
5.5	-.29659990	.49791844									
5.75	-1.6027873	-1.1089339									
6.0	-2.6558472	-2.5043784									

(a)  $I_2''$

$K_1$ $K_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.18339288	0.30620511	0.49032970	0.76613847	0.96234789	1.2231707	1.5906480	2.1621804	2.6027921	3.2651284	5.4458802
1.0	.27271803	.45620791	.72131948	1.1179138	1.3972517	1.7651034	2.2793165	3.0703096	3.6749891	4.5783072	7.5281264
1.25	.3842362	.64051024	1.0072396	1.5493419	1.9268792	2.4202381	3.1017899	4.1388567	4.9223502	6.084689	9.8243854
1.5	.51039267	.84222476	1.3323645	2.0415260	2.5317469	3.1679259	4.0390573	5.1844401	6.3300157	7.7716298	12.346538
1.75	.62480822	1.0431363	1.6425433	2.5246764	3.1343089	3.9237915	5.0002111	6.4778145	7.8007213	9.5448701	14.992268
2.0	.68380338	1.1552980	1.8439026	2.8759242	3.696754	4.5367846	5.8195923	7.7341760	9.1516915	11.206340	17.571830
2.25	.63393363	1.1033586	1.8205340	2.9421549	3.7473628	4.61245092	6.2673271	8.5057893	10.143014	12.530936	19.839662
2.5	.42966889	.81587140	1.4697519	2.5841448	3.4283685	4.5776593	6.2087075	8.5772218	10.573613	13.284396	21.565318
2.75	.05444864	.26412926	.74549531	1.7358821	2.5579965	3.7354791	5.4737462	8.2170199	10.296652	13.335361	22.612661
3.0	-.46081822	-.51047974	-.30541180	.43975954	1.1774527	2.3205451	4.1101212	7.0524050	9.3341008	12.697027	22.994280
3.25	-1.0278787	-1.3862536	-1.5303236	-1.1296985	-.53436265	.51476307	2.2956450	5.3915693	7.8552272	11.535792	22.875077
3.5	-1.5171740	-2.1740660	-2.6909308	-2.6966406		-1.3922100					
3.75	-1.7802575	-2.6678231	-3.5161519	-3.9472242		-3.0702968					
4.0	-1.6938917	-2.6826070	-3.7715072	-4.637774							
4.25	-1.2022123	-2.1327113	-3.3461105	-4.5525006							
4.5	-.34751823	-1.0624472	-2.2665586								
4.75	.72305098	.34522273	-.7320444								
5.0	1.7808348	1.8014247									
5.25	2.5665063	2.9785786									
5.5	2.8564111	3.5912186									
5.75	2.5283373	3.4881005									
6.0	1.6026281	2.6691602									

TABLE I.- VALUES OF THE INTEGRAL  $I_2$  FOR  $m = 2$  - Continued(e)  $I_2'$ 

$K_1 \backslash K_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.46312106	0.85314925	1.5397235	2.8703423	4.0532208	5.9666950	9.4460684	16.884283	28.858313	42.501668	164.18367
1.0	.35001004	.66352265	1.2397089	2.4028439	3.4670790	5.2239880	8.4538109	15.582354	23.296636	39.548196	160.95029
1.25	.18660481	.39078819	.80908243	1.7399713	2.6399713	4.1826096	7.1141932	13.808994	21.158433	36.890862	156.62355
1.5	-.03688781	.01879224	.22197348	.84270298	1.5272239	2.7882232	5.3324838	11.449955	18.347095	33.433058	151.08873
1.75	-.32196821	-.14561425	-.52102654	-.30142817	-.10825540	1.0129240	3.0707213	8.4509052	14.811909	29.104965	144.25586
2.0	-.65252004	-.10108462	-.1.3990557	-.1.6601725	-.1.5838798	-.1.1121083	-.35563580	1.566040	10.566040	23.908165	136.10717
2.25	-.98759159	-.15840423	-.2.3274066	-.3.1297746	-.3.4365995	-.3.4669224	-.2.6901851	-.77513680	5.7211316	17.953906	126.73608
2.5	-.1.2605924	-.2.0747088	-.3.1651692	-.4.5313174	-.5.2525992	-.5.8388793	-.5.8401693	-.3.5744996	-.51297860	11.4724609	116.35840
2.75	-.1.3885035	-.2.3554681	-.3.7343784	-.5.6321727	-.6.7723684	-.7.4426277	-.8.7887111	-.7.8161040	-.4.7347142	11.4787470	105.28467
3.0	-.1.2914650	-.2.3012329	-.3.8956195	-.6.1916103	-.7.7247720	-.9.4781236	-.11.208248	-.11.676887	-.9.6685200	1.7448730	93.857567
3.25	-.91918851	-.1.8277427	-.3.3946438	-.6.0257746	-.7.8939819	-.10.1977140	-.12.823040	-.14.786408	-.13.982138	-.7.8218220	82.374422
3.5	-.27656738	-.92842472	-.2.3358158	-.5.0685328	-.7.3821186	-.9.9755727	-.8.8442114				
3.75	.5601786	.29843246	-.7644988	-.3.3821186	-.1.2133618						
4.0	1.4450874	1.6584373	1.0653661	1.1093867							
4.25	2.1859229	2.8861738	2.8945399								
4.5	2.5861754	3.703822	4.3919734								
4.75	2.496870	3.8917756	5.0847095								
5.0	1.8650762	3.3502961									
5.25	.76170338	2.1371109									
5.5	-.61826673	.46600536									
5.75	-.1.9910399	-.1.3358455									
6.0	-.3.0461670	-.2.9023069									

(f)  $I_2''$ 

$K_1 \backslash K_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.44717227	0.76013840	1.3072519	2.1984602	2.9008868	3.9238038	5.5417987	8.4458416	11.140950	15.813365	38.335520
1.0	.59187144	1.0333730	1.7331848	2.9177451	3.8524957	5.2142727	7.3693369	11.292342	14.830490	21.059813	51.087407
1.25	.72320774	1.2657151	2.1290229	3.5963683	4.7569772	6.4512763	9.2136955	14.038765	18.449010	26.230489	63.757813
1.5	.82511119	1.4463584	2.4685668	4.1730304	5.5445233	7.5518645	10.752571	16.600230	21.888454	31.212325	76.224973
1.75	.85536645	1.5299779	2.6253920	4.568382	6.1019497	8.3958621	12.071452	18.444447	24.973905	35.820080	88.288913
2.0	.78547118	1.4422535	2.5438746	4.6003417	6.2846795	8.8024353	12.900133	20.516891	27.447147	39.807357	99.682304
2.25	.57481042	1.1354872	2.1743944	4.1972379	5.9349254	8.6094412	13.034813	21.426512	29.136040	42.906322	110.11136
2.5	.20456954	.57225612	1.4046422	3.2533040	4.9571903	7.6761985	12.322050	21.189953	29.753525	44.808683	119.31826
2.75	-.30941875	-.22959313	.2645144	1.7682450	3.3299340	5.9732043	10.698656	20.199627	29.219560	45.628246	127.14494
3.0	-.90540824	-.1.1830637	-.1.1379299	-.1.667815	1.1689026	3.5989766	8.2405783	18.024038	27.571251	45.111438	133.57331
3.25	-.1.4764003	-.2.1362541	-.2.4087576	-.2.2609844	-.1.3066002	.79026230	5.1956985	15.074752	24.990086	43.587515	138.72953
3.5	-.1.8861187	-.2.8849029	-.3.8735217	-.4.2621140	-.2.1209568	-.4.7696276					
3.75	-.1.9996212	-.3.2337994	-.4.6722162	-.5.8179973							
4.0	-.1.7280550	-.3.0266555	-.4.7873983	-.6.0555578							
4.25	-.1.0350486	-.2.2184455	-.4.1281744	-.6.6296553							
4.5	-.01880785	-.89538080	-.2.7601426								
4.75	1.1498911	.72487733	-.90540993								
5.0	2.2873039	2.31216175									
5.25	2.9559342	3.5890612									
5.5	3.1291000	4.2170299									
5.75	2.6501241	4.0628658									
6.0	1.5687406	3.1136252									

TABLE II.- VALUES OF THE INTEGRAL  $I_1$  FOR  $mB = 3$ (a)  $I_1$ 

$k_2 \backslash k_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.02125437	0.04908847	0.13118825	0.24376978	0.33188772	0.45647674	0.61352178	0.95267785	1.20159140	1.58937711	2.9347915
1.0	0.0796502	0.1022861	0.19029672	0.31122611	0.4118671	0.5223218	0.6537769	1.0251790	1.5131042	1.9481007	3.364985
1.25	0.08138694	0.1313510	0.2579999	0.4068735	0.5116715	0.6223218	0.7416110	1.1721319	1.9138938	2.4049323	3.9632810
1.5	0.0829548	0.1274710	0.2536855	0.47979307	0.65537561	0.89789042	1.2151157	1.7785622	2.1757183	2.7187550	4.4674512
1.75	0.08491440	0.115321	0.223903	0.42270651	0.6685477	0.8765114	1.2106223	1.623571	1.9228475	2.5197959	4.069080
2.0	0.11319195	0.25134237	0.3559988	0.42512213	0.39535772	0.48611769	0.6049805	0.8192847	0.7480537	1.5140236	3.1708066
2.25	0.11928896	0.25520114	0.3569751	0.333065	0.5020829	0.5896776	0.7136491	0.82053237	0.8412513	0.23295958	1.7726857
2.5	0.12643146	0.28383203	0.4029013	0.2112944	0.4794389	0.57977893	0.6875652	0.8853787	0.6177290	0.2700801	0.4776399
2.75	0.13047772	0.27089567	0.3173038	0.21930332	0.27062102	0.32205692	0.3417470	0.37996806	0.3480912	0.3190810	0.1823850
3.0	0.03941791	0.0852642	0.18392430	0.1818837	0.18160255	0.18568185	0.18628682	0.18628682	0.18628682	0.18628682	0.18628682
3.25	0.03941791	0.0852642	0.18392430	0.1818837	0.18160255	0.18568185	0.18628682	0.18628682	0.18628682	0.18628682	0.18628682
3.5	1.0095934	1.4976305	2.2263142	2.2263142	2.2263142	2.2263142	2.2263142	2.2263142	2.2263142	2.2263142	2.2263142
3.75	1.0310378	1.4601536	2.132651	1.7511364	1.7511364	1.7511364	1.7511364	1.7511364	1.7511364	1.7511364	1.7511364
4.0	1.4601536	1.094893	1.594893	1.7511364	1.7511364	1.7511364	1.7511364	1.7511364	1.7511364	1.7511364	1.7511364
4.25	1.55119789	1.1895325	1.5369510	1.2671066	1.2671066	1.2671066	1.2671066	1.2671066	1.2671066	1.2671066	1.2671066
4.5	1.5310839	1.0837063	1.4837063	1.6731063	1.6731063	1.6731063	1.6731063	1.6731063	1.6731063	1.6731063	1.6731063
4.75	1.4916649	0.7508879	1.28097385	1.28097385	1.28097385	1.28097385	1.28097385	1.28097385	1.28097385	1.28097385	1.28097385
5.0	1.2939938	0.2131568	0.2131568	0.2131568	0.2131568	0.2131568	0.2131568	0.2131568	0.2131568	0.2131568	0.2131568
5.25	1.1628254	0.5505875	0.5505875	0.5505875	0.5505875	0.5505875	0.5505875	0.5505875	0.5505875	0.5505875	0.5505875
5.5	1.6785911	1.1165290	1.1165290	1.1165290	1.1165290	1.1165290	1.1165290	1.1165290	1.1165290	1.1165290	1.1165290
5.75	2.5475855	2.6771148	2.6771148	2.6771148	2.6771148	2.6771148	2.6771148	2.6771148	2.6771148	2.6771148	2.6771148
6.0	2.1673948	2.6101263	2.6101263	2.6101263	2.6101263	2.6101263	2.6101263	2.6101263	2.6101263	2.6101263	2.6101263

(b)  $I_1$ 

$k_2 \backslash k_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00217913	0.00424061	0.00736819	0.01170869	0.01410641	0.01749127	0.02098767	0.02492336	0.02706381	0.02932352	0.03269291
1.0	0.01225594	0.0252829	0.04376202	0.06963064	0.09573668	0.121017467	0.14510992	0.16870750	0.19155827	0.2138112	0.2351133
1.25	0.01483936	0.03790356	0.0658731	0.10360055	0.14050893	0.17658373	0.211907	0.24651742	0.2804154	0.3133892	0.3454751
1.5	0.01550132	0.03792578	0.06325678	0.0930046	0.1230173	0.1530173	0.1830173	0.2130173	0.2430173	0.2730173	0.3030173
1.75	0.01771701	0.03572734	0.06268651	0.0925141	0.1217707	0.1512132	0.1808454	0.21061820	0.2404943	0.2704943	0.3004943
2.0	0.0206669	0.04234430	0.07764872	0.12125271	0.16876561	0.21290275	0.25679127	0.2976572	0.3349585	0.37038171	0.4049585
2.25	0.11506738	0.27112242	0.58159219	1.1306839	1.5360458	2.0559322	2.7151840	3.5263324	4.0296443	4.5715611	5.1319538
2.5	0.12863100	0.16310468	0.08776110	0.2481944	0.57512900	1.0660591	1.7606381	2.7184416	3.3161521	4.0054560	5.1141277
2.75	0.14613662	0.78377087	0.10833914	0.15592860	0.10113887	0.65811658	0.02115399	0.99084948	1.6915655	2.5272395	3.9767882
3.0	0.70507543	0.12774892	0.1946992	0.2464615	0.2464615	0.2464615	0.2464615	0.2464615	0.2464615	0.2464615	0.2464615
3.25	0.64205688	0.12621331	0.24073549	0.24073549	0.24073549	0.24073549	0.24073549	0.24073549	0.24073549	0.24073549	0.24073549
3.5	0.16223663	0.51395004	0.12447730	0.24073549	0.24073549	0.24073549	0.24073549	0.24073549	0.24073549	0.24073549	0.24073549
3.75	0.60809157	0.72473728	0.12447730	0.12447730	0.12447730	0.12447730	0.12447730	0.12447730	0.12447730	0.12447730	0.12447730
4.0	1.2993378	1.9339619	2.1380547	2.1380547	2.1380547	2.1380547	2.1380547	2.1380547	2.1380547	2.1380547	2.1380547
4.25	1.4638424	2.3917761	3.0009437	2.5792811	2.5792811	2.5792811	2.5792811	2.5792811	2.5792811	2.5792811	2.5792811
4.5	1.8456257	1.6915103	2.5185269	2.5185269	2.5185269	2.5185269	2.5185269	2.5185269	2.5185269	2.5185269	2.5185269
4.75	0.38680110	0.03389077	0.03389077	0.03389077	0.03389077	0.03389077	0.03389077	0.03389077	0.03389077	0.03389077	0.03389077
5.0	0.6664111	0.18018696	0.18018696	0.18018696	0.18018696	0.18018696	0.18018696	0.18018696	0.18018696	0.18018696	0.18018696
5.25	0.2722530	0.28572821	0.28572821	0.28572821	0.28572821	0.28572821	0.28572821	0.28572821	0.28572821	0.28572821	0.28572821
5.5	0.17515301	0.245474690	0.245474690	0.245474690	0.245474690	0.245474690	0.245474690	0.245474690	0.245474690	0.245474690	0.245474690
5.75	0.23580262	0.10229364	0.10229364	0.10229364	0.10229364	0.10229364	0.10229364	0.10229364	0.10229364	0.10229364	0.10229364
6.0	1.5577966	0.9032926	0.9032926	0.9032926	0.9032926	0.9032926	0.9032926	0.9032926	0.9032926	0.9032926	0.9032926

TABLE II.- VALUES OF THE INTEGRAL  $I_1$  FOR  $m = 3$  - Continued(a)  $I_2'$ 

$K_1$ $I_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.975
0.75	0.06167710	0.13830183	0.28868877	0.59567078	0.86951923	1.3037751	2.0518675	3.5120425	4.9720678	7.6246195	21.129987
1.0	.05600960	.18877222	.27257978	.57002133	.83794747	1.2651280	2.0058310	3.4873459	4.9126573	7.5604289	21.358163
1.25	.03696842	.08993033	.20486552	.46212267	.70495803	1.1036666	1.8113136	3.2558964	4.6610505	7.2875013	21.053355
1.5	-.01775337	-.01763323	.01754393	.16193115	.33365836	.66084647	1.269790	2.6035098	3.9905394	6.5153197	20.188633
1.75	-.12411990	-.22906197	-.35437786	-.44107892	-.41637699	-.27080592	.11530295	1.2562097	2.44775563	4.9082023	18.378778
2.0	-.27290978	-.52931110	-.89502710	-.1.3410388	-.1.5919840	-.1.6865074	-.1.6010225	-.87800390	-.12562510	2.3213024	15.432252
2.25	-.40227996	-.80791025	-.1.4321943	-.2.3009219	-.2.8072907	-.3.3089120	-.3.8729606	-.3.10977566	-.2.8136282	-.95689360	11.597535
2.5	-.46615833	-.86695377	-.1.6464161	-.2.8599995	-.3.5092193	-.4.5366280	-.5.1160885	-.5.9170154	-.5.6389683	-.4.2543402	7.5617961
2.75	-.18598286	-.51559821	-.1.2164205	-.2.5305467	-.3.5094779	-.4.7177087	-.6.1020586	-.7.3485713	-.7.5547747	-.6.7417081	4.1065777
3.0	.26414805	.27769311	-.05331859	-.1.1386893	-.2.1387153	-.3.5394662	-.5.3591151	-.7.3800451	-.8.1374856	-.8.0082378	1.6215291
3.25	.79408806	1.2719201	1.5260097	.98313730	.13015204	-.1.3054661	-.3.4524450	-.6.2412536	-.7.6072611	-.8.2750198	-.11.346590
3.5	1.1209501	1.9893913	2.8470939	3.0211396		1.1565360					
3.75	.96259620	1.9362598	1.1882959	4.0858265		2.9464504					
4.0	.22780807	.91332840	2.2057470	3.7118321							
4.25	-.85304971	-.77427311	.20493433	2.1375447							
4.5	-1.7693293	-2.3793992	-1.9512570								
4.75	-1.9672786	-3.0670035	-3.2854613								
5.0	-1.1772654	-2.3844228									
5.25	.36111498	-.56260602									
5.5	1.9442466	1.5539667									
5.75	2.7346872	2.9189162									
6.0	2.2263650	2.9862839									

(a)  $I_2''$ 

$K_1$ $I_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.975
0.75	0.05047470	0.10860928	0.21122800	0.40841651	0.56579647	0.79463387	1.1477424	1.7528035	2.2540958	3.0533840	5.9305802
1.0	.08012492	.17093011	.33336201	.62622877	.85970133	1.1946962	1.7039819	2.5620393	3.2634542	4.3699899	8.2886942
1.25	.12272871	.25992565	.50185020	.92958815	1.2665754	1.7379866	2.4449863	3.6111183	4.5477448	6.0043482	11.048519
1.5	.17115948	.36294482	.70058466	1.2937716	1.7543280	2.3989674	3.3488789	4.8868901	6.1010285	7.9607185	14.237196
1.75	.19576852	.4237133	.83635452	1.5798083	2.1648038	2.9880205	4.2018847	6.1551995	7.6819503	9.9936699	17.612805
2.0	.14457746	.34223448	.74035411	1.5280739	2.1837708	3.1367919	4.5779383	6.9311307	8.7828946	11.572937	20.624680
2.25	-.02911597	.02502860	.21704531	.87180075	1.4824639	2.4477557	4.0058232	6.6774982	8.8218822	12.084096	22.623622
2.5	-.31740431	-.52396858	-.65915469	-.46136665	-.03847987	.77447689	2.2823966	5.1193911	7.5012401	11.199496	23.251447
2.75	-.61750439	-1.1882084	-1.7228625	-2.1246430	-2.0468616	-1.9501530	-.27505372	2.5439252	5.0643404	9.1614984	22.727186
3.0	-.71512552	-1.4592317	-2.4445103	-3.1834432	-3.8132486	-3.7525005	-2.8765303	-.27226090	2.3295506	6.7055454	21.753790
3.25	-.52327961	-1.1896425	-2.3063999	-3.0115042	-4.5422149	-4.7886881	-4.5877964	-2.1782637	.05933590	4.6131054	21.049327
3.5	.08530669	-.22032278	-1.1042232	-2.7788025	-.6841186	-3.1452142					
3.75	.88509217	1.1605162	.82623226	-.6841186							
4.0	1.4810777	2.3279432	2.6794502	1.6065330							
4.25	1.4628350	2.6104877	3.5664275	3.1544097							
4.5	.66096298	1.6851345	3.0032385								
4.75	-.67775399	-1.6867328	1.2581676								
5.0	-1.9354905	-2.3088051									
5.25	-2.4118055	-3.1954991									
5.5	-1.7213102	-2.4570637									
5.75	-.06978269	-1.2509208									
6.0	1.7819760	.82000180									

TABLE II.- VALUES OF THE INTEGRAL  $I_1$  FOR  $m = 3$  - Continued

(a)  $I_1$

$\frac{K_1}{K_2}$	.4	.5	.6	.7	.75	.8	.85	.9	.95	.95	.95
0.75	0.09212288	0.22112139	0.50671731	1.1886519	1.8677359	3.0825310	5.4828331	11.215239	17.727785	32.086045	146.43489
1.0	0.1389362	.11811110	.30413977	.79938575	1.3392544	2.3413153	4.4233907	9.6099053	15.673330	29.319118	141.12364
1.25	-.03138911	-.04180519	-.00593522	.22151489	.54995420	1.2510099	2.8600163	7.3114825	12.763094	25.150819	133.90055
1.5	-.11286807	-.27566943	-.45681186	-.61120957	-.58004440	-.29353180	.71350330	4.1347043	8.7823935	20.228899	124.44147
1.75	-.28242634	-.57606725	-.40417572	-.17007497	-.20631109	-.23280330	-.21325680	-.01925100	3.6021718	13.487979	112.45916
2.0	-.41695662	-.87563577	-.16534746	-.28966674	-.37298495	-.4668916	-.54706102	-.49764684	-.26213545	5.3680060	98.112150
2.25	-.46820517	-.10296559	-.20516833	-.38399913	-.51567519	-.61263117	-.67601360	-.10159152	-.93042312	-.36018920	81.813892
2.5	-.34339566	-.85362180	-.19184012	-.40342528	-.57445825	-.80871798	-.11186941	-.14662400	-.15509217	-.12490104	64.584731
2.75	.03161705	-.22847853	-.10213267	-.30722187	-.49252943	-.71647117	-.11971085	-.17579383	-.20277476	-.20106493	47.229239
3.0	.53973261	.77039673	.58065169	.92529483	1.8929230	5.7917136	10.762571	18.417377	23.050707	26.057653	30.546402
3.25	1.0407366	1.8062465	2.4277780	3.8929230	4.4460046	5.7920036	7.8676177	17.327494	23.897331	30.261974	14.526806
3.5	1.2258906	2.3764729	3.7694001	4.4460046	5.7920036	7.8676177	10.762571	18.417377	23.050707	26.057653	30.546402
3.75	.86314740	2.0327786	3.8977735	5.7920036	7.8676177	10.762571	18.417377	23.050707	26.057653	30.546402	30.546402
4.0	-.05101231	.68854092	2.5485340	5.4276519	7.8676177	10.762571	18.417377	23.050707	26.057653	30.546402	30.546402
4.25	-.12028218	-.12348709	.12383449	3.5726489	7.8676177	10.762571	18.417377	23.050707	26.057653	30.546402	30.546402
4.5	-.20456516	-.29197863	-.24348759	4.0901767	7.8676177	10.762571	18.417377	23.050707	26.057653	30.546402	30.546402
4.75	-.20566943	-.35271006	-.40901551	4.0901767	7.8676177	10.762571	18.417377	23.050707	26.057653	30.546402	30.546402
5.0	-.10484512	-.5823043	1.7805466	3.3806483	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970
5.25	.64843296	1.7805466	3.3806483	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970
5.5	2.2728893	3.3806483	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970
5.75	2.9851909	3.3806483	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970
6.0	2.3851850	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970	3.5389970

(r)  $I_2$

$\frac{K_1}{K_2}$	.4	.5	.6	.7	.75	.8	.85	.9	.95	.95	.95
0.75	0.14006806	0.31370295	0.65391345	1.3471949	1.9649525	2.9438552	4.6291250	7.9643342	11.203138	17.173123	48.236430
1.0	.18508545	.41149852	.86605024	1.7869891	2.6065252	3.9112798	6.1555383	10.626031	14.916074	22.874779	64.289475
1.25	.22171763	.49990889	1.0501565	2.1821488	3.1971205	4.8118793	7.6016566	13.172248	18.525260	28.462925	80.216898
1.5	.23177947	.53266816	1.1434686	2.4321837	3.6065675	5.4912242	8.7048936	15.405043	21.793346	33.681267	95.730158
1.75	.18107584	.44707496	1.0256539	2.3463784	3.6020098	5.6696613	9.3565049	16.902282	24.258906	38.025598	110.26198
2.0	.03440088	.16602022	.56832869	1.6942741	2.8832417	4.9617069	8.8430631	17.089144	25.286354	40.794664	123.01024
2.25	-.22314005	-.34230339	-.31339585	.31975860	1.2416543	3.0855989	6.8683058	15.459773	24.296736	41.328392	133.18426
2.5	-.53779134	-.99096657	-.16993471	-.16572435	-.12249433	.09727170	3.4150273	11.889706	21.093420	39.343743	140.35534
2.75	-.77561008	-.15387713	-.26197713	-.37465378	-.39846400	-.31544732	-.98840800	6.8339215	16.068978	39.150286	144.67559
3.0	-.75789222	-.16514400	-.31370795	-.51867217	-.61763955	-.6339255	-.53750804	1.2227185	10.100396	29.542254	146.76362
3.25	-.35818813	-.10698197	-.25926201	-.52687504	-.6636262	-.69636262	-.87314000	3.9390505	4.1453342	23.398008	147.29171
3.5	.38253999	.18718534	-.91209795	-.37318887	-.98405765	-.98405765	-.98405765	-.98405765	-.98405765	-.98405765	-.98405765
3.75	1.8002833	1.7190611	1.4297412	1.4297412	1.4297412	1.4297412	1.4297412	1.4297412	1.4297412	1.4297412	1.4297412
4.0	1.6794624	2.8449279	3.5335961	2.0308349	4.2766075	4.2766075	4.2766075	4.2766075	4.2766075	4.2766075	4.2766075
4.25	1.4589288	2.9188795	1.7032708	3.8010915	1.7611710	1.7611710	1.7611710	1.7611710	1.7611710	1.7611710	1.7611710
4.5	.44056934	1.7032708	1.7611710	1.7611710	1.7611710	1.7611710	1.7611710	1.7611710	1.7611710	1.7611710	1.7611710
4.75	-.10900044	-.44787860	-.5464724	-.3704285	-.3170212	-.15568859	-.76055174	-.76055174	-.76055174	-.76055174	-.76055174
5.0	-.2588324	-.2588324	-.2588324	-.2588324	-.2588324	-.2588324	-.2588324	-.2588324	-.2588324	-.2588324	-.2588324
5.25	-.25935653	-.25935653	-.25935653	-.25935653	-.25935653	-.25935653	-.25935653	-.25935653	-.25935653	-.25935653	-.25935653
5.5	-.17044308	-.17044308	-.17044308	-.17044308	-.17044308	-.17044308	-.17044308	-.17044308	-.17044308	-.17044308	-.17044308
5.75	.12081151	.12081151	.12081151	.12081151	.12081151	.12081151	.12081151	.12081151	.12081151	.12081151	.12081151
6.0	2.0670697	.76055174	.76055174	.76055174	.76055174	.76055174	.76055174	.76055174	.76055174	.76055174	.76055174



TABLE III.- VALUES OF THE INTEGRAL  $I_1$  FOR  $mB = 4$ (a)  $I_1'$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00682117	0.01838656	0.04368909	0.09785398	0.14604437	0.22047526	0.34215192	0.54293125	0.75364012	1.0675619	2.2513744
1.0	.01190926	.03119580	.07288285	.15731575	.22919744	.33598401	.50255039	.78783773	1.0224427	1.3928923	2.7009490
1.25	.01763814	.04668185	.10928373	.23577920	.34209692	.51725597	.7328305	1.1183389	1.4216636	1.8803289	3.3738767
1.5	.01006680	.03160997	.08617382	.21513902	.33366084	.51557880	.80204693	1.2806386	1.6558772	2.2137968	3.9320784
1.75	-.03340448	-.07016378	-.11199858	-.11793613	-.07571059	.03253174	.26114778	.72242393	1.1184426	1.7314478	3.4379647
2.0	-.11034726	-.25819132	-.50005482	-.81981544	-.97933174	-1.0966215	-1.1028511	-.85429732	-.53708268	-.03553487	2.0213737
2.25	-.15192561	-.38097135	-.80128910	-1.4621315	-1.8788325	-2.2994794	-2.6647027	-2.7932762	-2.644101	-2.2115695	-.26232329
2.5	-.04483874	-.16439344	-.53254719	-1.2511559	-1.7901904	-2.1390413	-2.1321578	-3.6788767	-3.7548962	-3.5169658	-1.4950144
2.75	.22146542	.40304461	.48966921	.18672507	-.23439136	-.89403649	-1.7715411	-2.6995492	-3.0276153	-3.0526132	-1.4821626
3.0	.46627685	.99902540	1.6601007	2.0660262	1.9498841	1.4707215	.55112393	-.72818012	-1.3544439	-1.7352558	-.60978505
3.25	.36472587	.92733070	1.8418090	2.8118943	3.0414851	2.9893218	2.0232126	.44811422	-.44853110	-1.2983477	-.70962078
3.5	-.20354644	-.11596393	.44660486	1.5476476	1.5476476	2.2673762	.71747165				
3.75	-.87147824	-1.4929594	-1.6819239	-.8197742							
4.0	-.95390520	-1.9333618	-2.7539361	-2.3369468							
4.25	-.09781615	-.71190966	-1.7042679	-1.9680071							
4.5	1.1674769	1.4043372	.63232743								
4.75	1.6811570	2.6012845	2.2464684								
5.0	.71724939	1.6805778									
5.25	-1.1439941	-.66759292									
5.5	-2.2808419	-2.1063250									
5.75	-1.4933174	-2.0900559									
6.0	-.72694344	-.22298391									

(b)  $I_1''$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00035723	0.00097349	0.00181283	0.00336276	0.00441396	0.00574328	0.00732482	0.00921279	0.01068309	0.01144497	0.0132485
1.0	.00336277	.00823142	.01712042	.03183267	.04209410	.05453171	.06965578	.08776056	.09604917	.10922804	.12617098
1.25	.01546684	.03810556	.07970549	.14917861	.19770606	.25748775	.33025710	.41790956	.46795644	.52251417	.60702049
1.5	.01144491	.0309942	.07867714	.14592584	.19610503	.253102030	.319480599	1.2103685	1.3624550	1.5294661	1.7904895
1.75	.04338925	.14582758	.36513076	.72479549	.99121467	1.3336394	1.7689178	2.3166420	2.6441375	3.0005637	3.5712413
2.0	.03517547	.11055743	.29381010	.69171514	1.0218012	1.4809727	2.1048178	2.9441342	3.4622491	4.0571216	5.0369805
2.25	-.08681446	-.1406368	-.21006266	-.06307166	.13446194	.5232853	1.1635341	2.1572560	2.8266055	3.4365330	5.0495153
2.5	-.24751724	-.55207222	-.96570959	-1.4042812	-1.4828715	-1.3668317	-.95127522	.00317643	.75297927	1.7365969	3.5970047
2.75	-.26976276	-.66944865	-1.3575697	-2.2744687	-2.7052850	-2.9714562	-2.8715621	-2.1073488	-1.3373170	-2.21164763	2.1218969
3.0	.00623245	-.11957700	-.63755688	-1.6443841	-2.2412225	-2.8211968	-3.1078560	-2.6647351	-1.9589310	-.77236330	1.9982907
3.25	.48154083	.85444725	1.0446067	.49191075	-.13075718	-.92614298	-1.6446167	-1.1742304	-1.2569314	-.17316603	2.4425219
3.5	.73340780	1.5200941	2.3487311	2.4994496		1.1085918					
3.75	.34162711	.98779530	2.0038153	2.7310283		1.7181005					
4.0	-.59690746	-.66568078	-.05096769	1.0889009							
4.25	-1.3212920	-2.1600812	-2.2036503	-.81914479							
4.5	-.9894070	-2.4257649	-2.6219978								
4.75	.41666524	-.09695377	-1.0601776								
5.0	1.7953165	2.0039444									
5.25	1.7829524	2.5933872									
5.5	.10911860	.98335406									
5.75	-1.9044968	-1.2586121									
6.0	-2.4206969	-2.1896321									

TABLE III.- VALUES OF THE INTEGRAL  $I_2$  FOR  $nB = 4$  - Continued(c)  $I_2'$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.01289343	0.03711696	0.09637728	0.24359439	0.39302607	0.65264583	1.1436540	2.2217908	3.3348144	5.5219009	17.968301
1.0	0.0146125	0.0361364	0.08979693	0.23077292	0.37518048	0.62950661	1.1137193	2.1846071	3.2932863	5.4756556	17.914787
1.25	0.0139006	0.0317168	0.0644744	0.15049861	0.26987926	0.49260541	0.93865042	1.9632522	3.0456748	5.1994607	17.594402
1.5	-0.02438140	-0.05487150	-0.09680004	-0.19994984	-0.309017015	-0.52145302	-0.83130905	-1.1906700	-2.1782178	-4.2281806	-16.461536
1.75	-0.07919695	-0.19324777	-0.39548289	-0.69907088	-1.07257426	-1.6182797	-2.40305577	-3.57056880	-5.18364180	-7.9760421	-13.802637
2.0	-0.13576549	-0.34677686	-0.75486400	-1.4560718	-2.5388590	-4.2493439	-6.80465888	-9.92860253	-14.532823	-21.630923	-39.4163919
2.25	-0.11748908	-0.33871254	-0.83446330	-1.8280790	-3.2019389	-5.36711573	-8.3036502	-11.9754757	-17.21150	-25.6327503	-48.259250
2.5	0.05243449	0.22260157	0.23734826	0.9996426	1.9483920	3.2094877	4.9659900	7.1269042	9.808735	13.091440	21.7691710
2.75	0.31100015	0.67557683	1.0011439	1.8117619	2.8672899	4.3346808	6.2624902	8.5944027	11.310530	14.563845	22.968683
3.0	0.48942952	1.1433041	2.1035228	2.9248965	3.5990015	4.1063475	4.507732	4.8503924	5.1456384	5.3970969	5.628770
3.25	0.24828478	0.80494690	1.9534622	3.5990015	4.1063475	4.507732	4.8503924	5.1456384	5.3970969	5.628770	5.8924088
3.5	-0.39956979	-0.45346929	-1.7825163	1.9726052	-0.8284549	3.9436189	2.3667526	-1.1779688	-4.2834158	-7.9470969	-12.9268770
3.75	-0.99795680	-1.8243611	-2.3677939	-0.8284549	-2.8383728	-3.9436189	-5.1456384	-6.3970969	-7.7410530	-9.1836418	-10.7163919
4.0	-0.89212410	-0.54307927	-2.0300941	-2.9183542	-3.7065736	-4.507732	-5.3970969	-6.3970969	-7.410530	-8.563845	-9.824088
4.25	1.25966211	1.7211785	2.57065736	3.57065736	4.507732	5.3970969	6.3970969	7.410530	8.563845	9.824088	11.1836418
4.5	1.3681277	2.8804698	4.8259280	7.1269042	9.808735	12.563845	15.310530	18.056323	20.702923	23.259250	25.726377
4.75	1.7311831	3.70360160	6.2624902	9.3036502	12.563845	15.310530	18.056323	20.702923	23.259250	25.726377	28.1994607
5.0	0.55899145	1.8259280	3.5990015	4.1063475	4.507732	4.8503924	5.1456384	5.3970969	5.628770	5.8924088	6.1456384
5.25	-1.3816351	-2.4319445	-3.70360160	-5.1456384	-6.3970969	-7.410530	-8.563845	-9.824088	-11.1836418	-12.563845	-14.056323
5.5	-2.4319445	-3.70360160	-5.1456384	-6.3970969	-7.410530	-8.563845	-9.824088	-11.1836418	-12.563845	-14.056323	-15.628770
5.75	-1.8789941	-2.8539670	-3.70360160	-4.507732	-5.3970969	-6.3970969	-7.410530	-8.563845	-9.824088	-11.1836418	-12.563845
6.0	-0.84524592	-1.56782287	-2.3677939	-3.2019389	-4.1063475	-5.07732	-6.03924	-7.002923	-8.066323	-9.1336418	-10.200969

(d)  $I_2''$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.01296305	0.03570001	0.08726200	0.20285369	0.30980866	0.48052169	0.77033211	1.1200665	1.6123846	2.2476075	5.5636804
1.0	0.02207119	0.05997147	0.14404033	0.32734148	0.509125	0.75272939	1.1436540	1.9838919	2.6868111	3.8616306	8.4202508
1.25	0.03713879	0.09982959	0.23613291	0.52508292	0.7797542	1.1698150	1.8012251	2.9340496	3.9041970	5.4911966	11.4445448
1.5	0.05306056	0.14401747	0.34354313	0.76732917	1.1367718	1.7021008	2.5992093	4.1685078	5.495809	7.5758716	15.135351
1.75	0.04905321	0.13369530	0.36912591	0.84970341	1.3116217	2.0317908	3.1996059	5.2527635	6.9563051	9.6443919	19.023052
2.0	-0.02513756	-0.0706851	-0.05261691	-0.0246377	0.1515780	1.5426549	2.8366700	5.2618914	7.3310940	10.620850	21.956229
2.25	-0.16418141	-0.36284743	-0.62504249	-0.77524885	-0.6501453	-1.19387242	-1.91873541	-3.4189614	-5.7192186	-9.520828	-22.831528
2.5	-0.28025303	-0.60689907	-1.3507085	-2.2328484	-2.599267	-2.6959009	-2.1215977	-0.0347070	-2.3766343	-6.5195130	-21.723994
2.75	-0.20914888	-0.59564349	-1.4290766	-2.6543589	-3.7232624	-4.1918652	-4.7157098	-3.3013444	-1.1546794	3.0990897	80.013474
3.0	0.15618795	0.3238009	0.32068141	1.7017249	-2.8499535	-4.1992395	-5.3312907	-5.0230430	-3.3520930	7.0696800	18.986137
3.25	0.61169778	1.1756540	1.5552531	0.89160146	-1.1977429	-1.8901045	-3.9093703	-5.0263425	-4.1375423	-6.6221230	18.150548
3.5	0.72395895	1.6513584	2.8192469	3.2292813	3.2292813	3.2292813	3.2292813	3.2292813	3.2292813	3.2292813	3.2292813
3.75	0.16705213	0.82403710	2.1915462	3.6054774	1.8848630	2.3388005	2.3388005	2.3388005	2.3388005	2.3388005	2.3388005
4.0	-0.82007208	-1.0160350	-1.8366826	-1.8848630	-1.8848630	-1.8848630	-1.8848630	-1.8848630	-1.8848630	-1.8848630	-1.8848630
4.25	-1.4168918	-2.4699830	-2.5849561	-3.1481094	-3.1481094	-3.1481094	-3.1481094	-3.1481094	-3.1481094	-3.1481094	-3.1481094
4.5	-0.87102530	-2.4279763	-1.9715660	-1.9715660	-1.9715660	-1.9715660	-1.9715660	-1.9715660	-1.9715660	-1.9715660	-1.9715660
4.75	0.6636690	0.24846905	-1.9715660	-1.9715660	-1.9715660	-1.9715660	-1.9715660	-1.9715660	-1.9715660	-1.9715660	-1.9715660
5.0	1.9862011	2.3448641	2.3448641	2.3448641	2.3448641	2.3448641	2.3448641	2.3448641	2.3448641	2.3448641	2.3448641
5.25	1.7867638	2.8929931	2.8929931	2.8929931	2.8929931	2.8929931	2.8929931	2.8929931	2.8929931	2.8929931	2.8929931
5.5	-0.05463046	1.2202953	1.2202953	1.2202953	1.2202953	1.2202953	1.2202953	1.2202953	1.2202953	1.2202953	1.2202953
5.75	-2.1122416	-1.2098003	-1.2098003	-1.2098003	-1.2098003	-1.2098003	-1.2098003	-1.2098003	-1.2098003	-1.2098003	-1.2098003
6.0	-2.5470653	-2.4263295	-2.4263295	-2.4263295	-2.4263295	-2.4263295	-2.4263295	-2.4263295	-2.4263295	-2.4263295	-2.4263295

TABLE III. - VALUES OF THE INTEGRAL  $I_2$  FOR  $mB = k$  - Continued(c)  $I_2'$ 

$K_2$	$K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.01651833	0.05288302	0.15443967	0.45772183	0.81115150	1.5169372	3.0688027	7.1906959	12.269533	24.150066	128.58068	
1.0	-0.0057349	-0.0574270	-0.122539	-0.1975351	-0.22193089	-0.20717910	-0.03764510	-0.03764510	-0.03764510	-0.03764510	-0.03764510	
1.25	-0.02968224	-0.07282118	-0.1573645	-0.22193089	-0.22193089	-0.20717910	-0.03764510	-0.03764510	-0.03764510	-0.03764510	-0.03764510	
1.5	-0.07537638	-0.19593710	-0.43731614	-0.86951829	-1.1667076	-1.4715153	-1.5682380	-1.5682380	-1.5682380	-1.5682380	-1.5682380	
1.75	-0.12805125	-0.34325548	-0.80010683	-1.7056516	-2.3908454	-2.8258096	-3.3784180	-4.0123096	-4.7111514	-5.4711514	-6.2311514	
2.0	-0.14421903	-0.41247962	-1.0337092	-2.3908454	-2.8258096	-3.3784180	-4.0123096	-4.7111514	-5.4711514	-6.2311514	-7.0000000	
2.25	-0.05271885	-0.22812730	-0.77335600	-2.2592306	-3.7236792	-6.0371119	-9.6548624	-15.057984	-22.280568	-32.261192	-45.557710	
2.5	-0.17619348	-0.30987063	-0.3956644	-0.73828700	-1.1996998	-1.83109030	-2.6644359	-3.8033690	-5.2602595	-7.0000000	-9.0000000	
2.75	-0.4265832	-0.97751281	-1.7185187	-2.6926999	-4.3634068	-6.8711907	-10.810061	-16.610565	-24.150066	-34.150066	-47.150066	
3.0	-0.8704216	-1.2798904	-2.0915370	-3.222623	-4.8320456	-6.8711907	-10.810061	-16.610565	-24.150066	-34.150066	-47.150066	
3.25	-0.9485769	-1.62042747	-2.89895068	-4.5912712	-6.8711907	-10.810061	-16.610565	-24.150066	-34.150066	-47.150066	-62.150066	
3.5	-1.230915	-2.2353666	-4.0977977	-6.8711907	-10.810061	-16.610565	-24.150066	-34.150066	-47.150066	-62.150066	-80.150066	
3.75	-1.1230915	-2.2353666	-4.0977977	-6.8711907	-10.810061	-16.610565	-24.150066	-34.150066	-47.150066	-62.150066	-80.150066	
4.0	-0.80403990	-2.1772889	-3.9937458	-3.8260818	-4.5288359	-5.5912712						
4.25	-0.38652487	-2.3131770	-2.5206788	-4.5288359								
4.5	1.6377429	2.3168259	3.371578									
4.75	1.7856625	3.3000173	3.099217									
5.0	3.7929119	2.0373758										
5.25	-1.6647931	-0.7840424										
5.5	-2.6270747	-2.9641916										
5.75	-1.4768679	-2.9622681										
6.0	1.0400009	-0.98575664										

(d)  $I_2''$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.03899175	0.1121464	0.29071793	0.73372649	1.1829557	1.9629577	3.4367638	6.6734225	10.013419	16.575692	53.916411
1.0	0.05110966	0.1480694	0.38468170	0.97283907	1.5700690	2.6079507	4.5704364	8.8629048	13.394893	22.082291	71.866978
1.25	0.05970665	0.17392586	0.45761320	1.1727846	1.9054175	3.1857035	5.6181663	10.983990	16.534391	27.165385	89.661078
1.5	0.05126411	0.15853769	0.44005850	1.2103952	2.0252777	3.4823481	6.3046819	12.627130	19.221005	32.251659	106.78773
1.75	0.0046098	0.03986201	0.21041875	0.82718750	1.5808836	3.0350222	6.0245927	13.030566	20.509806	35.477677	122.06684
2.0	-0.10838261	-0.23466276	-0.37030401	-0.25254317	-0.17855880	-0.12914673	-0.08167140	-0.05152937	-0.03156045	-0.0195270	-0.01156045
2.25	-0.24524566	-0.60052224	-1.2071167	-1.9591600	-2.1659682	-2.5836655	-3.1671140	-4.0169740	-5.0977439	-6.4597439	-8.24661
2.5	-0.29143099	-0.79149112	-1.8066781	-3.5348091	-4.5784918	-5.6030918	-6.7088762	-7.9016714	-9.2731300	-10.848769	-12.71942769
2.75	-0.16163309	-0.45281792	-1.4578688	-3.7432761	-5.5318920	-7.6088762	-9.9178762	-12.4816714	-15.202694	-18.0715620	-21.1089503
3.0	-0.33190290	-0.90015961	-2.5564384	-6.8101815	-9.9257617	-12.9257617	-15.9257617	-18.9257617	-21.9257617	-24.9257617	-27.9257617
3.25	-0.74100938	-1.5591726	-3.3237276	-6.6966881	-10.2239220	-13.7117408	-17.1516990	-20.5436990	-23.8836990	-27.1736990	-30.4136990
3.5	-0.68704837	-1.7917751	-3.9029194	-7.5469499	-10.810061	-14.0616714	-17.2616714	-20.4136990	-23.5136990	-26.5636990	-29.5636990
3.75	-0.64448302	-1.60845118	-3.4795127	-6.1359429	-9.2137788	-12.3778788	-15.5137788	-18.6137788	-21.6637788	-24.6637788	-27.6637788
4.0	-1.0688867	-1.4634329	-3.39728755	-6.1639037	-9.176458	-12.176458	-15.176458	-18.176458	-21.176458	-24.176458	-27.176458
4.25	-1.5136637	-2.8817766	-5.2137788	-8.1176458	-11.0616714	-14.0136990	-16.9636990	-19.9136990	-22.8636990	-25.8136990	-28.7636990
4.5	-1.72900387	-2.2818429	-4.9767412	-7.9767412	-10.9767412	-13.9767412	-16.9767412	-19.9767412	-22.9767412	-25.9767412	-28.9767412
4.75	-1.94900628	-2.18612675	-4.2954036	-6.2954036	-9.2954036	-12.2954036	-15.2954036	-18.2954036	-21.2954036	-24.2954036	-27.2954036
5.0	-2.2127638	-2.7261067	-3.3324292	-4.0136990	-4.6949499	-5.3761999	-6.0574499	-6.7386999	-7.4199499	-8.1011999	-8.7824499
5.25	-1.8013804	-2.3324292	-3.3324292	-4.0136990	-4.6949499	-5.3761999	-6.0574499	-6.7386999	-7.4199499	-8.1011999	-8.7824499
5.5	-2.4691874	-2.8024732	-3.45292540	-4.10292540	-4.75292540	-5.40292540	-6.05292540	-6.70292540	-7.35292540	-8.00292540	-8.65292540
5.75	-2.732315	-2.8024732	-3.45292540	-4.10292540	-4.75292540	-5.40292540	-6.05292540	-6.70292540	-7.35292540	-8.00292540	-8.65292540
6.0	-2.7239216	-2.7239216	-3.45292540	-4.10292540	-4.75292540	-5.40292540	-6.05292540	-6.70292540	-7.35292540	-8.00292540	-8.65292540

TABLE IV.- VALUES OF THE INTEGRAL  $I_1$  FOR  $m = 6$ (a)  $I_1'$ 

$K_1$ $K_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00032643	0.00142502	0.00512868	0.01678068	0.030010942	0.05473900	0.10307132	0.20947836	0.31590211	0.51292989	1.4174019
1.0	.00078074	.00329903	.01133641	.04476388	.09961618	.10336480	.18277192	.34251829	.48068276	.74776959	1.8065998
1.25	.00134943	.00509591	.02084267	.06509589	.11216362	.19245712	.33356707	.59967831	.83000275	1.8025448	2.5385233
1.5	.00161447	.00643318	.00578540	.01330180	.04930829	.12856445	.29430845	.64407598	.95930380	1.6685868	3.1740204
1.75	.01288490	.04768494	.13523357	.30492130	.61829809	.92862594	1.58188578	2.44149976	3.19741548	4.2940188	2.2172507
2.0	.01470978	.06357400	.22509647	.62913627	1.7766700	3.4444098	5.9959989	8.4667285	10.790964	12.3881880	1.69521702
2.25	.03150473	.08758194	.13957893	.00572142	.29132153	.82965070	1.6898579	2.8167558	3.3522575	3.6893835	2.4411329
2.5	.08035627	.29112428	.76355561	1.4529585	1.7103121	1.6855407	1.1219932	1.19212518	1.0815810	1.9073069	2.4999274
2.75	.02320693	.02302266	.36454208	1.3497701	2.0677891	2.7052709	2.8250371	1.7996878	.72044573	-.59949404	-1.2414508
3.0	.02910796	.0457275	-1.2183810	-1.2143651	-.63903480	.30037587	1.2103785	1.0644752	.17678267	-1.3139671	-2.8391194
3.25	.08073951	.45732783	-1.4237837	-2.5431116	-2.5450803	-1.7417236	-.21394026	.93079819	.63755106	-.61020800	-2.7248214
3.5	.0051189	.91460057	.97606712	-.21351088	-.97186882	-.53490927					
3.75	.35397673	1.2178232	2.3546301	1.440576							
4.0	-.56995081	-.80600222	.01218096	.97240045							
4.25	-.81964063	-2.0109618	-2.2596550	-.10287494							
4.5	.57099948	.21180822	-.89092001								
4.75	1.4022103	2.3687744	1.0535050								
5.0	-.31181989	.60023245									
5.25	-1.9861784	-1.9665712									
5.5	-.20588851	-1.0891913									
5.75	2.1795067	1.0147404									
6.0	.8367644	.82612345									

(b)  $I_1''$ 

$K_1$ $K_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00000985	0.00003785	0.00011293	0.00028586	0.00043332	0.00063936	0.00092150	0.00130076	0.00153423	0.00180176	0.00223999
1.0	.00083174	.00068825	.00266805	.00678162	.01030416	.01524805	.02204757	.03123186	.03691278	.04342994	.05191904
1.25	.00181123	.00702805	.02142279	.05533668	.08483310	.12672588	.18506044	.26888825	.3476415	.43726484	.56832375
1.5	.00554498	.02234495	.07116066	.19300086	.30364989	.46291747	.69938425	1.0296830	1.4811803	1.9000327	1.9120574
1.75	.00250385	.01547766	.06782729	.23945215	.42288968	.72184932	1.1959154	1.9297188	2.4294506	3.0416858	4.1301242
2.0	-.02252003	-.07484820	-.17685034	-.27763267	-.25981226	-.10675035	1.30911346	1.1932345	1.9049440	2.8448430	4.7534301
2.25	-.03926726	-.15638634	-.46811588	-1.0912876	-1.5023045	-1.8849778	-2.0450723	-1.6028591	-.95661580	-.12464408	2.6946125
2.5	.03437636	.06340060	-.02999874	-.58052928	-1.1620745	-1.9514621	-2.7760475	-3.1216102	-2.7674500	-1.8194609	1.2567630
2.75	.14044431	.46144438	1.0429912	1.5284450	1.4116331	.82275592	-.27485219	-1.4410481	-1.6236708	-1.0714568	2.1176855
3.0	.01173943	.19317858	.85145568	2.0930024	2.6242245	2.6443879	1.7395059	-.06170430	-.8866444	-1.0345609	1.7801598
3.25	-.31202801	-.80993162	-1.2096877	-.56250373	.27593112	1.0550120	1.0039643	-.58731653	-1.7590286	-2.4873641	-.07536343
3.5	-.19286290	-.80820597	-1.9687218	-2.5021476		-.4519403					
3.75	.50175503	.92185031	.53909398	-.81220156		-.07702729					
4.0	.56483725	1.6304428	2.4696441	1.0369397							
4.25	-.59819498	-.56222677	.53908435	.59759171							
4.5	-1.1053725	-2.2716171	-1.7520843								
4.75	.47705959	-.19799530	-.95146421								
5.0	1.6690635	2.2730588									
5.25	-.07958129	.91995747									
5.5	-2.0985207	-1.5453440									
5.75	-.52148128	-1.0619873									
6.0	2.1545344	.47804128									

TABLE IV.- VALUES OF THE INTEGRAL  $I_2$  FOR  $mB = 6$  - Continued(c)  $I_2'$ 

$K_2$	$K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00056369	0.00267235	0.01073420	0.04070382	0.08022835	0.16339915	0.3281375	0.87357810	1.4993864	2.8961402	12.779896	
1.0	0.00044752	0.00222868	0.00940457	0.03733079	0.07510994	0.15583318	0.3138754	0.85812277	1.4811321	2.8746744	12.753148	
1.25	0.00079358	0.00256030	0.00510592	0.00010671	0.0125775	0.07124272	0.22087072	0.68280400	1.2732680	2.6292661	12.445585	
1.5	0.0062969	0.0203913	0.07199103	0.17690308	0.25667981	0.36511653	0.39583654	0.21298189	0.2079280	1.3504990	10.820228	
1.75	0.0133765	0.05952021	0.19521764	0.54123428	0.85481245	1.3022637	1.8924970	2.5062509	2.6131360	2.0995335	6.2737707	
2.0	0.02211667	0.02614369	0.11880231	0.60160470	1.1094500	1.9507888	3.2725118	5.1640560	6.2116156	6.8913105	7.5474060	
2.25	0.05048354	0.16589592	0.37200094	0.7204377	1.2407084	2.01911813	2.81243238	4.9706788	7.0722580	9.3161965	9.5434061	
2.5	0.05030363	0.25535539	0.67622140	1.5485527	2.5826079	3.8735899	5.5761718	8.6995919	12.2593564	16.0570841	19.1000951	
2.75	0.071404761	0.3066405	1.0285691	1.3372170	2.5224738	3.8735899	5.5761718	8.6995919	12.2593564	16.0570841	19.1000951	
3.0	0.23067569	0.75302347	1.5997031	1.8011119	3.2058538	4.85676687	7.2158503	10.699773	15.9132330	23.9264405	36.074200	
3.25	0.00223873	0.30158572	1.1320888	1.3598289	1.49030322	2.3396338	3.8938723	5.9909259	9.5527100	14.72934260	22.1343977	
3.5	0.07572251	1.1584323	1.3598289	1.49030322	2.3396338	3.8938723	5.9909259	9.5527100	14.72934260	22.1343977	36.074200	
3.75	0.26287137	1.1818080	2.7115594	1.8938723	5.9909259	9.5527100	14.72934260	22.1343977	36.074200	54.171899	88.969015	
4.0	0.70173760	1.0895298	0.3906069	1.8938723	5.9909259	9.5527100	14.72934260	22.1343977	36.074200	54.171899	88.969015	
4.25	0.76361430	2.1586316	1.8938723	5.9909259	9.5527100	14.72934260	22.1343977	36.074200	54.171899	88.969015	141.71220	
4.5	0.75772230	3.5463862	1.8938723	5.9909259	9.5527100	14.72934260	22.1343977	36.074200	54.171899	88.969015	141.71220	
4.75	1.4222295	2.6174882	7.2934260	14.72934260	22.1343977	36.074200	54.171899	88.969015	141.71220	229.33082	368.969015	
5.0	0.89519405	2.6174882	7.2934260	14.72934260	22.1343977	36.074200	54.171899	88.969015	141.71220	229.33082	368.969015	
5.25	2.0308423	2.6174882	7.2934260	14.72934260	22.1343977	36.074200	54.171899	88.969015	141.71220	229.33082	368.969015	
5.5	0.9933082	2.6174882	7.2934260	14.72934260	22.1343977	36.074200	54.171899	88.969015	141.71220	229.33082	368.969015	
5.75	2.3282231	0.88969015	1.3994411	2.2963554	2.0706553	1.8165825	1.5770268	1.3788835	1.2068524	1.0831245	0.9183142	
6.0	0.86171220	1.1994411	2.2963554	2.0706553	1.8165825	1.5770268	1.3788835	1.2068524	1.0831245	0.9183142	0.7608524	

(d)  $I_2''$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00078032	0.00351754	0.01319088	0.04953337	0.08444828	0.15958700	0.31474801	0.67777530	1.0593331	1.7965822	5.4380306
1.0	0.00155392	0.00682604	0.02476365	0.06197189	0.11826798	0.27233426	0.51982432	1.0768680	1.6452756	2.7176405	7.8192862
1.25	0.00323949	0.01399614	0.04948897	0.15753901	0.27747507	0.49311551	0.90300606	1.7732163	2.6202637	4.1573224	11.037324
1.5	0.00391846	0.01837997	0.07031466	0.23915495	0.43181814	0.78021377	1.4357866	2.7641940	4.0475915	6.2511867	15.411395
1.75	0.00614453	0.01718405	0.02273798	0.05706405	0.21362901	0.57009682	1.3530576	3.1243338	4.8385768	7.8282574	19.746979
2.0	0.03066667	0.11480880	0.32285234	0.69587121	0.89431396	0.96932674	0.6423681	0.92257260	2.7796677	6.3192067	21.029360
2.25	0.02480591	0.12397278	0.46068531	1.3391490	2.0784436	2.9758982	3.7524077	3.4566341	2.0733555	1.3966050	18.346021
2.5	0.06802082	0.30309894	0.25139408	0.24281777	1.0422547	2.4227744	4.3411243	5.9368710	5.6407086	3.0101744	15.189657
2.75	0.13638275	0.50366880	1.3002334	2.2146570	2.2040547	1.3090391	0.96547834	4.3785958	5.5860475	4.5820524	13.454894
3.0	0.05650771	0.24339669	0.69132968	2.4728631	3.5038449	3.8502796	2.4416689	2.0345545	4.7799075	5.9443527	10.417897
3.25	0.34701544	0.98958110	1.6451438	0.78949139	0.69177230	2.4407684	2.8047761	0.81770502	4.4743016	7.7233127	5.9009733
3.5	0.12126976	0.69785069	2.1693376	3.1860490	1.5370268	1.6275060					
3.75	0.60151321	1.2051303	0.75878732	1.5370268		0.93788835	0.73011134				
4.0	0.89540033	1.6947694	2.8893501								
4.25	0.76368283	0.75859323	0.8341245	1.3698524							
4.5	1.0837602	2.4699397	1.9183142								
4.75	0.6706982	0.18374710									
5.0	1.7492655	2.4699062									
5.25	0.2349086	1.1816585									
5.5	0.2333642	1.5692631									
5.75	0.47654015	1.4602340									
6.0	2.2963554	0.20706553									

TABLE IV.- VALUES OF THE INTEGRAL  $I_2$  FOR  $MB = 6$  - Continued

(a)  $I_2'$

$K_2$	$K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00028280	0.00186912	0.01085657	0.05557340	0.13078345	0.32189750	0.86333750	2.7109055	5.5168733	13.076384	98.418164	
1.0	-0.00136342	-0.00553997	-0.01685608	-0.03671832	-0.03621186	-0.00697540	-0.25218720	1.4496009	3.5208457	9.7343094	88.517471	
1.25	-0.00486166	-0.02063120	-0.07065333	-0.21040572	-0.34736415	-0.51989690	-0.70598790	-0.63890080	-0.3862430	-0.6643351	71.543295	
1.5	-0.01076119	-0.04675189	-0.16519723	-0.51895571	-0.88991102	-1.5151099	-2.5149185	-4.0451442	-4.5863676	-3.0875192	54.831445	
1.75	-0.01125353	-0.05594357	-0.22550138	-0.79986270	-1.4623052	-2.6515992	-4.8010650	-8.6796836	-11.467032	-13.866084	28.285610	
2.0	-0.01584776	-0.0912914	-0.3518398	-1.34163930	-1.0619621	-2.5306454	-5.6212933	-12.108351	-17.637878	-24.990550	-2.8466670	
2.25	-0.06628475	-0.21717946	-0.7115975	-1.8588890	-1.2951557	-3.5096430	-8.2855957	-10.275550	-18.319154	-31.080915	-32.720404	
2.5	-0.1359376	-0.4304463	-1.5035140	-2.7275820	-4.0221890	-5.0744878	-4.3257201	-2.7219928	-12.093687	-29.469973	-57.681319	
2.75	-0.22719672	-0.32212235	-2.8670778	-1.2339239	3.2229420	6.1093757	8.6313113	5.3989786	-3.3277663	-23.523710	-79.111038	
3.0	-0.22719600	-0.84832620	-2.0869826	-2.7768787	-1.5840937	1.7734873	7.2845780	10.056661	1.3297374	-15.778391	-97.331387	
3.25	-0.10077927	-0.09108801	-1.1158295	-0.8984094	-5.0973468	-3.3221321	-5.1021784	12.167739	11.356022	-5.3947515	-109.54601	
3.5	-0.53251893	1.4402867	1.9089164	-0.6136871	1.0939319	-4.5913364						
3.75	-1.1848007	1.1224231	3.2366062	1.0868480	3.2480477							
4.0	-0.84581680	-1.4439566	-0.8668480	1.1097042								
4.25	-0.68832544	-2.3529697	-3.1866698									
4.5	-0.96430802	-5.4369476	-2.1036376									
4.75	1.4408510	2.9690411	.88031862									
5.0	-0.70623250	.88854601										
5.25	-2.1590400	-2.3742143										
5.5	-0.2382474	-1.9053662										
5.75	2.5212685	.77529166										
6.0	.89601070	1.6714470										

(c)  $I_2''$

$K_2$	$K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00254876	0.01207837	0.04846710	0.18358125	0.36166040	0.73623020	1.5980088	3.9330062	6.7404792	13.035251	57.512689	
1.0	.00334563	.01590103	.06400996	.24322365	.47998838	.97815970	2.1256564	5.2369090	8.9909265	17.370495	76.671108	
1.25	.00337637	.01677786	.07066644	.28016704	.56347957	1.1687794	2.5788955	6.4350776	11.107199	21.558182	95.645577	
1.5	-0.0116558	-0.0021316	.02239432	.17381163	.4629150	1.0277010	2.5449056	6.9315878	12.387082	24.752996	113.36561	
1.75	-0.11707624	-0.06390402	-0.17782491	-0.35416942	-0.38224505	-0.15992710	.92691550	5.0675394	10.804213	24.169016	126.61093	
2.0	-0.3420309	-0.14531086	-0.46198190	-1.3086208	-1.9435587	-2.7905121	-3.1116637	-8.82049610	4.0671212	17.718213	130.87251	
2.25	-0.0140394	-0.05695112	-0.37233806	-1.5645320	-2.6701892	-4.8938888	-7.5295555	-8.9585383	-6.4256967	5.0356900	124.99970	
2.5	.10144522	.32284267	.64565800	.37157718	-0.68655390	-3.1304232	-7.6634715	-13.742724	-14.969264	-8.0913064	112.93006	
2.75	.11791174	.51675996	1.5883895	3.2762818	3.6365825	2.4279893	-2.2266734	-12.175335	-17.914757	-17.701447	97.528463	
3.0	-0.13466661	-0.19545092	.43314665	3.0417101	5.0557612	6.3274478	3.9369987	-7.2845082	-17.077206	-24.732718	77.632822	
3.25	-0.36917833	-1.1811284	-2.2392455	-1.2268162	1.3138220	4.9958931	6.7774732	-1.8885693	-13.677863	-29.094605	58.070827	
3.5	.01073849	-0.50177832	-2.44443213	-4.3715100	1.8505513	-1.8925112						
3.75	.69951236	1.5482091	1.0943690	-2.6560292								
4.0	.36280291	1.7601186	3.5327177	.87559957								
4.25	-0.4375980	-1.1017569	1.2249376	2.5724690								
4.5	-1.0497041	-2.7888293	-2.2299249									
4.75	.69129020	-0.15769496	-2.4023451									
5.0	1.8239083	2.8326640										
5.25	-0.41179109	1.5139386										
5.5	-2.4036374	-1.6507470										
5.75	-0.2668736	-1.9695491										
6.0	2.4688880	-1.10313628										

TABLE V.- VALUES OF THE INTEGRAL  $I_1$  FOR  $m = 8$ (a)  $I_1'$ 

$\frac{K_1}{K_2}$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00001616	0.00011571	0.00063291	0.00392219	0.00652132	0.01128270	0.02639820	0.04193012	0.13912320	0.25870569	0.92995562
1.0	0.00005287	0.0036023	0.0183717	0.0799168	0.14822639	0.2342735	0.3499000	0.5168311	0.8127250	1.1611835	1.2699382
1.25	0.0000780	0.0072284	0.0369718	0.1846909	0.3803102	0.6771385	1.0837344	1.6352557	2.3693507	3.359886	1.9691684
1.5	0.00009220	0.01140168	0.07140168	0.2756145	0.53223108	0.9189807	1.4671625	2.251131	3.351131	4.95470450	2.616453
1.75	0.00193988	0.0275367	0.1972311	0.7157053	1.3737694	2.3737694	3.9013763	5.9013763	8.9013763	13.9013763	21.9013763
2.0	0.0035846	0.0668050	0.429271	1.859503	3.852266	6.852266	10.852266	16.852266	25.852266	39.852266	60.852266
2.25	0.0131163	0.2688088	1.8793555	8.9523451	18.35206	31.51297	49.1213708	72.1213708	105.1213708	155.1213708	235.1213708
2.5	0.02694885	0.54681625	3.82235242	18.2235242	38.2235242	68.2235242	108.2235242	168.2235242	258.2235242	398.2235242	608.2235242
2.75	0.0458859	0.9195119	6.5546003	32.7546003	65.7546003	115.7546003	185.7546003	285.7546003	435.7546003	665.7546003	1015.7546003
3.0	0.0812720	1.62929131	11.558117	57.706647	115.706647	215.706647	365.706647	565.706647	865.706647	1315.706647	2015.706647
3.25	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964
3.5	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964
3.75	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964
4.0	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964
4.25	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964
4.5	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964
4.75	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964
5.0	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964
5.25	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964
5.5	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964
5.75	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964
6.0	0.08571724	1.71313818	12.4075612	62.2957071	124.591414	229.182828	390.331381	627.218971	1045.19584	1718.65964	2618.65964

(b)  $I_1''$ 

$\frac{K_1}{K_2}$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00000048	0.00000210	0.00000702	0.00002164	0.00004309	0.00007256	0.00011835	0.00018812	0.00023376	0.00029017	0.00036859
1.0	0.00001607	0.0000760	0.000312	0.0017302	0.0035665	0.00635152	0.01128270	0.02135623	0.04120270	0.0766234	0.1275049
1.25	0.00002658	0.00127280	0.0068043	0.02035295	0.03618663	0.06217515	0.10366149	0.16830618	0.2653932	0.4090716	0.6094796
1.5	0.00007751	0.00394885	0.01976285	0.07972519	0.15068263	0.27462519	0.46516803	0.73588799	1.0867379	1.6017639	2.369463
1.75	0.00160939	0.0076775	0.02111943	0.06151686	0.10379163	0.17759950	0.2913738	0.46516803	0.73588799	1.0867379	1.6017639
2.0	0.00497007	0.0205136	0.06269577	0.16430538	0.27901382	0.46516803	0.73588799	1.0867379	1.6017639	2.369463	3.536511
2.25	0.0132091	0.04710974	0.14010219	0.3538581	0.59791251	0.9620151	1.46516803	2.20516803	3.3333333	4.95470450	7.2676314
2.5	0.026376	0.1175860	0.3503518	0.798094	1.3123483	2.076855	3.1655336	4.7333333	6.8888889	9.7777778	13.9013763
2.75	0.0513674	0.23056595	0.6668144	1.59571024	2.65711857	4.121292	6.102961	8.7325979	12.5000000	17.7777778	25.5555556
3.0	0.09305174	0.39763880	1.17563160	2.8636113	4.7333333	7.2676314	10.801667	15.801667	22.5000000	31.6666667	44.4444444
3.25	0.17939217	0.70561190	1.9581197	4.8235580	7.901382	12.4075612	18.618809	27.3333333	39.5833333	56.6666667	79.0123457
3.5	0.31664222	1.25052187	3.5070260	8.932800	14.591414	22.9182828	35.7333333	52.7777778	77.7777778	111.1111111	155.5555556
3.75	0.51240068	1.93421561	5.4219260	13.505011	22.9182828	35.7333333	52.7777778	77.7777778	111.1111111	155.5555556	218.7500000
4.0	0.78924595	2.781077	7.731077	18.010543	28.010543	42.010543	60.010543	84.010543	116.010543	166.010543	236.010543
4.25	1.0534216	3.9650377	11.27577802	26.18987	39.2847229	57.4270714	82.618809	116.010543	166.010543	236.010543	333.3333333
4.5	1.3558765	5.3144005	15.4585768	36.18987	54.2847229	80.4270714	116.010543	166.010543	236.010543	333.3333333	466.6666667
5.0	1.6022303	6.4022303	18.70295778	44.4444444	66.6666667	100.0000000	150.0000000	225.0000000	337.5000000	511.1111111	740.7407407
5.25	1.8972271	7.4022303	21.81489148	51.1111111	76.6666667	115.0000000	172.5000000	258.7500000	388.1250000	582.1875000	843.2777778
5.5	2.0652456	7.9223627	23.223627	55.5555556	83.3333333	125.0000000	187.5000000	281.2500000	421.8750000	632.8125000	916.6666667
5.75	2.1692544	8.1607187	24.1607187	57.7777778	86.6666667	130.0000000	195.0000000	292.5000000	438.7500000	658.1250000	960.0000000
6.0	2.2222222	8.3333333	24.6666667	58.8888889	88.8888889	133.3333333	197.7777778	296.6666667	444.4444444	666.6666667	977.7777778

TABLE V.- VALUES OF THE INTEGRAL  $I_2$  FOR  $n_0 = 8$  - Continued(c)  $I_2'$ 

$K_1$ $K_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.965
0.75	0.00002447	0.00019232	0.00119526	0.00679581	0.01634473	0.04068060	0.11002992	0.34330792	0.67364292	1.5180970	9.0281204
1.0	.00001503	.00013864	.00096170	.00599940	.014677308	.03853594	.10619820	.33721332	.66623040	1.5086392	9.0154220
1.25	-.00015040	-.00087254	-.01860209	-.00961032	-.01295065	-.00912289	.02729549	.81003313	.50622520	1.3084612	8.7439203
1.5	-.00100299	-.00635415	-.02922462	-.00718644	-.01159565	-.03246310	-.51144636	-.69682512	-.65666600	-.17457470	6.6774364
1.75	-.00074907	-.00736471	-.04748533	-.23268732	-.47540919	-.92712480	-1.7260393	-3.0271292	-3.4421879	-4.4871497	-1.5941510
2.0	.00695749	.03686745	.12440466	.23539875	.17629197	-.18627979	-1.2539709	-3.7436316	-5.8118577	-8.4191456	-8.2589892
2.25	.00498402	.04766713	.27346394	1.0462297	1.7424472	2.5005705	2.7319474	3.6741565	4.7278510	5.8068448	-11.894690
2.5	-.03113825	-.16119190	-.42960368	-.36803658	-.30060045	1.7728558	3.6741565	4.7278510	5.8068448	6.9939955	-13.808513
2.75	-.00184812	-.10700103	-.71764776	-2.3139323	-1.0688443	-2.8443527	-4.7611645	-3.3683939	3.8764108	4.4838621	-15.032967
3.0	.11511138	.51507104	1.18495488	6.3744426	8.3228016	2.5575380	2.3571838	2.2764408	5.1304032	5.9931927	-14.097744
3.25	-.07528496	-.00958082	1.0085741	2.9133390	2.6438972	1.2903698	2.5160163	1.44700181	3.5449663		
3.5	-.24148704	-1.0970023	-2.2332187	-.61389184		2.6025064					
3.75	.35683102	.64859601	-.58620037	-1.8730989							
4.0	.23386046	1.1346158	2.5471315	-.61345962							
4.25	-.85975230	-1.7377165	-.04984271	.30404477							
4.5	.23821684	-.02690590	-1.5108312								
4.75	1.2093445	2.3683516	-.33578173								
5.0	-1.2330454	-.20790761									
5.25	-.78668006	-1.7661630									
5.5	2.1001024	.54431197									
5.75	-.47196623	.71409900									
6.0	-1.9353459	.34663723									

(d)  $I_2''$ 

$K_1$ $K_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.965
0.75	0.00004464	0.00032778	0.00188015	0.00962247	0.02165209	0.04679935	0.12067706	0.32600076	0.57933953	1.1387782	4.5989519
1.0	.000010552	.00071827	.00407989	.01952264	.04232975	.09322866	.21509594	.54892967	.94406290	1.7876445	6.7332284
1.25	.00027785	.00193619	.01022973	.04641452	.09655313	.20605561	.43652224	1.0265494	1.6674131	2.9629366	9.8739615
1.5	.00002769	.00092811	.00903565	.05901382	.13282338	.32027849	.73527310	1.7570929	2.8345824	4.830664	14.626475
1.75	-.00278923	-.01606413	-.06340127	-.17449872	-.23007258	-.25695196	-.02367685	1.1350267	2.5385121	5.3749657	18.651013
2.0	-.00245474	-.02807996	-.13018303	-.55511882	-1.0227272	-1.7242136	-2.5551006	-2.8326235	-2.0246844	-.66224200	16.730160
2.25	.01605792	.07958596	.23497592	.29226978	-.01637277	-.92500090	-2.7869654	-5.4366978	-6.3139736	-5.2827110	11.388837
2.5	.04456826	.04107170	.47912527	1.6707461	2.5009396	2.9607705	1.9920129	-1.7461686	-4.44840286	-6.1790679	8.3003507
2.75	-.06607098	-.30172980	-.74387205	-.448770195	.59467615	2.4080093	3.8009037	1.5837287	-2.0311820	-6.4597853	3.3652142
3.0	.01994720	.79417172	.92203390	2.7916257	-3.1783677	-1.9076080	1.2747136	2.8258210	.20494933	-5.6152687	-1.7111846
3.25	.17841462	.79417172	1.7237768	.72926352	-1.0154220	-2.1300185	-.06758949	4.4429320	3.9591941	-1.7673625	-4.8144548
3.5	-.18309411	-.24890163	.68931850	2.5821343		-1.3595356					
3.75	-.27444954	-1.3437086	-2.5558475	-1.6308037		-.30942063					
4.0	.59291909	1.1781817	2.2086880	-1.0228007							
4.25	.07321178	1.8897687	2.1632400	-1.5013712							
4.5	-1.0938511	-2.1852174	-.04694727								
4.75	.69350937	-.3396996	-.82310619								
5.0	1.1226354	2.3203924									
5.25	-1.7469029	-.54833785									
5.5	-.22193597	-1.1956744									
5.75	2.1787056	.19502181									
6.0	-1.1352263	.5629469									



TABLE V.- VALUES OF THE INTEGRAL  $I_p$  FOR  $m = 6$  - Continued

(e)  $I_5'$

$K_1$ $K_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	-0.00164372	-0.00008458	-0.00005907	0.00333739	0.00388015	0.00399896	0.0037170	0.95182600	2.3108331	6.7046050	72.276875
1.0	-0.00077735	-0.00008108	-0.00002761	-0.02422401	-0.04694714	-0.08397459	-0.11556840	-1.0055800	-8.2175050	-3.3297830	-61.049804
1.25	-0.00142222	-0.00362343	-0.01313249	-0.08644244	-0.17813128	-0.34373471	-0.73647380	-1.7094273	-1.7015975	-6.1903905	-13.456116
1.5	-0.00142603	-0.00774720	-0.04333743	-0.20619517	-0.43654105	-0.91494540	-1.9392664	-4.2025746	-7.5752107	-12.123965	-19.453592
1.75	-0.001483	-0.00390872	-0.0612362	-0.15633187	-0.45946880	-1.1098844	-3.0218956	-6.6399700	-13.287508	-26.380030	-50.000245
2.0	-0.001983	-0.05252789	-0.2041240	-0.63423384	-1.6006326	-3.76135286	-8.6997700	-19.3171215	-41.211359	-97.443365	-109.65852
2.25	-0.00159374	-0.0299492	-0.1053044	-0.3047186	-0.7992718	-1.905611	-4.7993424	-10.325849	-23.603464	-53.3583778	-110.72052
2.5	-0.00100299	-0.02021166	-0.08234886	-0.2722869	-0.7286284	-1.807513	-4.7553512	-10.325849	-23.603464	-53.3583778	-110.72052
2.75	-0.00187432	-0.01404473	-0.05148984	-0.1541153	-0.46027176	-1.2856580	-3.771552	-1.5601382	11.453315	23.512726	-104.29281
3.0	-0.1627526	-0.59229093	-1.6130413	-3.9127542	-8.6691970	-19.3171215	-41.211359	-97.443365	-109.65852	-110.72052	-104.29281
3.25	-0.12960721	-0.20081403	-0.89958490	-2.7195875	-6.2370149	-14.744225	-3.0190440	-1.0046744	1.0046744	1.0046744	1.0046744
3.5	-0.21169777	-1.161483	-3.28880000	-6.2370149	-14.744225	-3.0190440	-1.0046744	1.0046744	1.0046744	1.0046744	1.0046744
3.75	-0.3739924	-1.6590505	-4.1028998	-7.147425	-1.0046744	1.0046744	1.0046744	1.0046744	1.0046744	1.0046744	1.0046744
4.0	-0.13783995	-0.24623029	-0.8469884	-2.1195163	-0.8469884	-2.1195163	-0.8469884	-2.1195163	-0.8469884	-2.1195163	-0.8469884
4.25	-0.9254770	-2.6725380	-6.725380	-1.4172213	-0.6699884	-2.1195163	-0.8469884	-2.1195163	-0.8469884	-2.1195163	-0.8469884
4.5	-0.2097305	-0.6699884	-2.1195163	-0.8469884	-2.1195163	-0.8469884	-2.1195163	-0.8469884	-2.1195163	-0.8469884	-2.1195163
4.75	-1.4172213	-0.6699884	-2.1195163	-0.8469884	-2.1195163	-0.8469884	-2.1195163	-0.8469884	-2.1195163	-0.8469884	-2.1195163
5.0	-0.72212978	-0.37540741	-0.37540741	-0.37540741	-0.37540741	-0.37540741	-0.37540741	-0.37540741	-0.37540741	-0.37540741	-0.37540741
5.25	-2.2582417	-1.7194156	-1.7194156	-1.7194156	-1.7194156	-1.7194156	-1.7194156	-1.7194156	-1.7194156	-1.7194156	-1.7194156
5.5	-0.51676497	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249
5.75	-2.0940869	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249
6.0	-2.0940869	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249	-0.45536249

(r)  $I_5''$

$K_1$ $K_2$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00122169	0.00115736	0.00118137	0.0062302	0.09827055	0.24522081	0.46041230	2.0602275	4.0435233	9.1091553	54.169392
1.0	-0.00166344	-0.0151626	-0.0546231	-0.05403597	-0.13033496	-0.32606991	-0.7864900	-2.7139500	-5.3875994	-12.140663	-72.219629
1.25	-0.00000698	-0.00115949	-0.0067713	-0.05624278	-0.14304510	-0.37382427	-1.0426959	-3.3407455	-6.6225968	-15.038447	-90.085899
1.5	-0.00104669	-0.00507011	-0.01909321	-0.06001905	-0.02153876	-0.11290549	-0.68526740	-3.0790631	-6.7658280	-14.551206	-106.05020
1.75	-0.00423389	-0.02254826	-0.10870491	-0.49077754	-0.7223231	-1.1604957	-1.5318089	-2.4092250	-4.022247	-7.8895317	-105.240742
2.0	-0.00005260	-0.00204081	-0.07546305	-0.56387477	-1.2902874	-2.7013483	-5.1222053	-11.577324	-16.328319	-25.682113	-84.187262
2.25	-0.00061083	-0.011232	-0.0168046	-0.0855302	-0.3087253	-1.0914560	-3.9380047	-11.577324	-16.328319	-25.682113	-84.187262
2.5	-0.00680199	-0.0280070	-0.3926121	-2.0291541	-3.5519077	-6.9094633	-12.820104	-25.682113	-40.887113	-60.887113	-84.187262
2.75	-0.0159696	-0.3646847	-1.0670305	-1.1161283	-3.9246035	-7.669523	-14.717967	-25.682113	-40.887113	-60.887113	-84.187262
3.0	-0.05773878	-0.4599541	-1.0670305	-3.9246035	-7.669523	-14.717967	-25.682113	-40.887113	-60.887113	-60.887113	-84.187262
3.25	-0.16848102	-0.87303270	-2.2057691	-9.6949182	-14.5867354	-24.7262430	-44.6123598	-79.263760	-114.71389	-144.71389	-184.71389
3.5	-0.25326977	-1.49066228	-3.3369090	-3.7086351	-3.7086351	-3.7086351	-3.7086351	-3.7086351	-3.7086351	-3.7086351	-3.7086351
3.75	-0.2148049	-1.3093341	-3.0919536	-3.5163460	-3.5163460	-3.5163460	-3.5163460	-3.5163460	-3.5163460	-3.5163460	-3.5163460
4.0	-0.7323417	-1.4762823	-3.7136440	-2.0341270	-2.0341270	-2.0341270	-2.0341270	-2.0341270	-2.0341270	-2.0341270	-2.0341270
4.25	-0.06042559	-1.3066853	-2.7462294	-2.5361849	-2.5361849	-2.5361849	-2.5361849	-2.5361849	-2.5361849	-2.5361849	-2.5361849
4.5	-1.1311167	-2.4837774	-3.8501730	-1.4265790	-1.4265790	-1.4265790	-1.4265790	-1.4265790	-1.4265790	-1.4265790	-1.4265790
4.75	-0.87606542	-1.41182427	-1.41182427	-1.41182427	-1.41182427	-1.41182427	-1.41182427	-1.41182427	-1.41182427	-1.41182427	-1.41182427
5.0	-1.0658345	-2.6987174	-2.6987174	-2.6987174	-2.6987174	-2.6987174	-2.6987174	-2.6987174	-2.6987174	-2.6987174	-2.6987174
5.25	-1.9196272	-3.8910408	-3.8910408	-3.8910408	-3.8910408	-3.8910408	-3.8910408	-3.8910408	-3.8910408	-3.8910408	-3.8910408
5.5	-1.14481738	-1.6356247	-1.6356247	-1.6356247	-1.6356247	-1.6356247	-1.6356247	-1.6356247	-1.6356247	-1.6356247	-1.6356247
5.75	-2.3290132	-0.4875298	-0.4875298	-0.4875298	-0.4875298	-0.4875298	-0.4875298	-0.4875298	-0.4875298	-0.4875298	-0.4875298
6.0	-1.2051848	-1.0502710	-1.0502710	-1.0502710	-1.0502710	-1.0502710	-1.0502710	-1.0502710	-1.0502710	-1.0502710	-1.0502710

TABLE VI.- VALUES OF THE INTEGRAL  $I_1$  FOR  $m\beta = 9$ (a)  $I_1'$ 

$\frac{x_1}{x_2}$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.0000047	0.0000315	0.0002232	0.0012953	0.0030682	0.0073809	0.0185854	0.0518485	0.0934158	0.1658521	0.7612109
1.0	0.0000155	0.0001217	0.0007472	0.0036511	0.0085226	0.0182532	0.0427431	0.1046329	0.1758447	0.3139710	1.048914
1.25	0.0002399	0.0026575	0.0173221	0.0696307	0.2224759	0.6948115	1.1107759	2.5326661	3.9755995	6.5787777	1.7471506
1.5	0.0027699	0.0196456	0.0900513	0.2866239	0.6124764	1.0491879	1.6183886	2.4182293	3.4783284	4.7515420	2.3767841
1.75	0.0046861	0.0345342	0.1306103	0.3714960	0.7915835	1.3367046	2.0612720	3.092543	4.4788346	6.1912940	1.6787060
2.0	0.0325184	0.2215754	0.8406937	1.9121987	3.8958240	6.565597	10.5369280	16.7105104	25.308206	38.315549	3.0019673
2.25	0.0746111	0.4691701	1.8441359	4.5875347	9.1139575	15.7954473	25.3189098	38.908424	57.651187	85.3812088	1.6839355
2.5	0.1307934	0.7163909	2.8505105	7.0277061	13.669006	23.567266	36.818169	54.804852	81.603512	120.277	2.3715604
2.75	0.209778	1.161729	4.7043715	11.6375966	22.5261637	38.261227	57.651187	85.381208	120.277	174.3324	2.4819964
3.0	0.3184049	1.8271185	7.0104174	17.233213	28.086968	45.777917	68.181699	100.65876	148.6555	217.00501	1.3059126
3.25	0.4562544	2.6392065	10.135598	25.6570916	39.560088	59.383940	87.99968	131.02727	194.2238	284.4779	1.9302396
3.5	0.6271521	3.5912061	13.861028	34.649012	52.60012	78.069142	113.77917	170.65876	254.6555	374.3324	2.4819964
3.75	0.8366374	4.9151574	19.055964	47.347250	71.347250	104.4442	154.4442	224.4442	334.4442	504.4442	2.4819964
4.0	1.0866374	6.639975	25.6570916	64.649012	92.60012	134.4442	194.4442	284.4442	424.4442	634.4442	2.4819964
4.25	1.3866374	8.7128291	34.649012	87.347250	121.347250	174.4442	254.4442	374.4442	554.4442	814.4442	2.4819964
4.5	1.7366374	11.245396	46.649012	117.347250	161.347250	234.4442	344.4442	514.4442	754.4442	1104.4442	2.4819964
4.75	2.1366374	14.345396	61.649012	157.347250	211.347250	314.4442	464.4442	684.4442	1014.4442	1484.4442	2.4819964
5.0	2.5866374	18.045396	79.649012	207.347250	271.347250	394.4442	574.4442	844.4442	1244.4442	1814.4442	2.4819964
5.25	3.0866374	22.345396	100.649012	267.347250	341.347250	484.4442	704.4442	1034.4442	1514.4442	2184.4442	2.4819964
5.5	3.6366374	27.245396	124.649012	337.347250	411.347250	584.4442	844.4442	1244.4442	1814.4442	2614.4442	2.4819964
5.75	4.2366374	32.745396	152.649012	417.347250	491.347250	694.4442	1004.4442	1454.4442	2114.4442	3064.4442	2.4819964
6.0	4.8866374	38.945396	184.649012	507.347250	581.347250	814.4442	1174.4442	1684.4442	2414.4442	3464.4442	2.4819964

(b)  $I_1''$ 

$\frac{x_1}{x_2}$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	-0.00000151	-0.00000126	0.00000056	0.00000681	0.00001370	0.00002530	0.00004521	0.00007674	0.00009724	0.00012416	0.00017501
1.0	0.00000510	0.00003214	0.00016836	0.00068783	0.00189314	0.0033605	0.00607451	0.0089179	0.0086874	0.01133979	0.01583912
1.25	0.0007076	0.0053963	0.0291266	0.1230970	0.2359317	0.4035126	0.7758975	1.1342953	1.7490845	2.2636122	3.2166723
1.5	0.0015515	0.0148738	0.0958879	0.4859085	1.0166313	1.8044903	3.054065	4.754065	7.1157073	1.0043662	2.0239001
1.75	0.0011538	0.00674661	0.04804302	0.2702355	0.5778546	1.0222728	1.8050308	2.8353680	4.250657	6.225502	8.3050475
2.0	0.0066372	0.0319610	0.06887583	0.33507915	0.61093392	1.0963465	1.6195266	2.1886294	3.0187200	4.225532	5.766603
2.25	0.0048751	0.02551750	0.11860192	0.4860893	0.9363481	1.6074547	2.4619930	3.598718	5.192545	7.2371118	9.8666762
2.5	0.00964174	0.04778441	0.10560202	0.76497402	1.5096806	2.3200517	3.5488619	5.2657877	7.5996974	10.9077244	1.1732803
2.75	0.02371136	0.11859730	0.78502570	1.7206115	3.17370602	4.94794930	7.2934509	10.686241	15.57288	22.457288	1.1223046
3.0	0.07674961	0.35098167	0.69116969	0.8694654	1.0421445	1.5205346	2.2894682	3.5348647	5.1616014	7.466332	1.54427935
3.25	0.14597072	0.2743443	0.8700273	1.1698871	1.6906462	2.4879454	3.7710738	5.6561013	8.3732470	12.1670538	1.7422664
3.5	0.24578776	0.45702523	2.0137744	1.0195281	1.6056111	2.3797556	3.5496885	5.2657877	7.5996974	10.9077244	1.1732803
3.75	0.34939546	0.6362838	0.68995612	0.6056111	0.35581173	0.70954418	1.0963465	1.6195266	2.1886294	3.0187200	4.225532
4.0	0.46150633	0.22220040	1.1600081	0.35581173	0.70954418	1.0963465	1.6195266	2.1886294	3.0187200	4.225532	5.766603
4.25	0.5866121	1.7931152	1.16057627	0.70954418	1.0963465	1.6195266	2.1886294	3.0187200	4.225532	5.766603	7.2371118
4.5	0.7899710	1.634256	0.6702148	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559
4.75	0.55590871	0.30689116	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559
5.0	0.78382059	1.6907078	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559
5.25	1.7678049	1.0301778	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559
5.5	1.1285140	1.9011074	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559
5.75	0.7434313	0.04361828	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559
6.0	1.9720823	0.7302138	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559	0.01643559

TABLE VI.- VALUES OF THE INTEGRAL  $I_2$  FOR  $MB = 9$  - Continued.(c)  $I_2'$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00000577	0.00005135	0.00039715	0.00277236	0.00738832	0.02083796	0.06125758	0.21519182	0.45170119	1.0990559	7.5836291
1.0	0.00006561	0.00039527	0.00305560	0.0239223	0.0665721	0.1711386	0.39902803	0.81138267	1.4680544	1.9928013	7.5749065
1.25	-0.00005777	-0.00022124	-0.00211979	-0.00770431	-0.0253095	-0.05978234	-0.10308961	-0.1766836	-0.30870760	-0.5167610	7.3237106
1.5	-0.0036907	-0.00300285	-0.01715106	-0.07640048	-0.1676106	-0.27965020	-0.49301995	-0.77884459	-1.06517908	-1.63734390	5.0539732
1.75	0.0022389	-0.0002576	-0.0280047	-0.1146107	-0.28957437	-0.4998310	-0.71521576	-0.93808099	-1.02774642	-0.9999307	-2.1161788
2.0	0.0059777	0.02629984	0.12737726	0.40563965	0.9070470	1.61136156	2.4169204	3.2191312	3.91777521	-7.6347287	-10.888847
2.25	-0.0497390	-0.0381044	0.01735134	0.0090403	1.2201220	2.40148590	3.7277426	5.125323	6.4883974	-3.0971111	-13.071786
2.5	-0.1329523	-0.1121869	-0.51028522	-1.1760486	-1.7375404	-1.0999372	1.1912560	4.3229436	4.3674762	7.3121260	-15.20890
2.75	0.0491881	0.0453907	0.387068	0.9984940	2.7864984	-3.3127225	-2.9772269	1.5102363	4.2797888	3.7223954	-15.360252
3.0	-0.1137485	0.0951473	0.9984940	-1.1478768	-2.70206514	1.7263640	-2.1115172	-4.5839012	3.5247323	6.4646688	-12.985961
3.25	-0.2128618	-0.6965649	-1.1478768	-2.1577790	-2.1577790	1.8782472	-3.0990372	-3.8765369	-6.3321945	5.2420797	-11.043181
3.5	0.21349466	0.9184621	2.1420605	4.6767115	1.0610955	-0.57069408					
3.75	-0.0325102	0.6815352	1.4831513	1.3106752							
4.0	-0.0184450	-1.7963263	-1.0441172	0.96518170							
4.25	-0.7718364	0.89776930	0.6516405								
4.5	-0.0833360	1.3947224									
4.75	-1.1217723	-2.205250									
5.0	1.4442215	0.5546135									
5.25	-1.17698517	1.1934914									
5.5	-1.6111283	-0.73214042									
5.75	2.0217719	-5.2400711									
6.0	-0.0084583	0.0480585									

(d)  $I_2''$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00000813	0.00006645	0.00069956	0.00436899	0.01082766	0.02746559	0.07374493	0.22301245	0.42245678	0.89361600	4.1630867
1.0	0.00028951	0.0024636	0.0164659	0.0947400	0.2241855	0.5401218	1.13681332	2.38710688	4.0587124	1.42899290	6.1513702
1.25	0.00008289	0.0072182	0.0469845	0.2523345	0.57004655	1.2926922	3.0863861	7.7163460	1.3189311	2.4761963	9.1983047
1.5	-0.0008845	-0.0030395	0.0122714	0.2342355	0.6998979	1.9205791	5.0851246	1.3726128	2.3430683	4.2707095	14.058477
1.75	-0.00125014	-0.0062833	-0.0474624	-0.18379906	-0.30596935	-1.3615844	-2.8253227	-2.8300529	1.4421743	4.0631692	17.695140
2.0	0.0112182	0.0227965	-0.0274406	-0.28591515	-0.68783614	-1.4526131	-2.6705964	-3.9244422	-3.8703776	-2.0181612	13.860969
2.25	0.0790151	0.030532	0.28521353	0.82396710	1.0540467	0.82402708	0.9994390	3.9471269	-5.9813962	-6.7205116	8.0792751
2.5	-0.0451730	-0.07391717	-0.09376356	0.2067108	1.6884576	3.1672637	3.9875735	1.46399670	-1.9336501	-5.8305439	4.3242070
2.75	-0.1253420	-0.15155462	-0.8354818	-2.2384698	-2.4312710	-1.1964113	1.6682030	2.9442011	2.9766605	-5.4420162	-2.0621116
3.0	0.8227431	0.2609621	1.021971	0.6820634	-1.5379640	-2.7065251	-0.8902496	3.8295677	3.8400009	-1.4661257	-5.6346906
3.25	-0.0828677	-0.11707250	0.80273903	2.7708859	2.2246034	-4.2973315	-2.4360638	1.7881473	4.5281162	1.9897560	-9.7813532
3.5	-0.11490614	-0.85226375	-2.3405377	-0.95308280							
3.75	0.37737092	1.2390269	0.77966846	-1.3885884							
4.0	-2.7860748	0.0687493	1.8645530	-0.50594775							
4.25	-0.30498125	-1.9113082	-1.5379890	1.3303642							
4.5	1.0408677	1.0231797	-0.56516203								
4.75	-0.7043363	0.0333532	0.79419247								
5.0	-0.71387563	-1.9427741									
5.25	1.8597532	0.7434542									
5.5	-1.2505126	-0.7302379									
5.75	-0.7683454	-2.28449176									
6.0	2.0997690	-0.77161517									

TABLE VI.- VALUES OF THE INTEGRAL  $I_9$  FOR  $m = 9$  - Continued(e)  $I_9'$ 

$\frac{k_1}{k_2}$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	-0.00097773	-0.00060775	-0.00028504	-0.00022181	0.00246823	0.01734024	0.09328460	0.53817300	1.4553474	4.7174513	61.366949
1.0	-0.00106041	-0.00041075	-0.00270882	-0.01474727	-0.03273008	-0.06964792	-0.13774080	-0.12654450	-0.1905870	-2.1561116	-19.882580
1.25	-0.00177906	-0.0013427	-0.00931786	-0.05103952	-0.11585823	-0.26224398	-0.59875820	-1.3661833	-1.9631793	-2.0895870	-32.909292
1.5	-0.00106481	-0.00279910	-0.02026968	-0.12101142	-0.28403143	-0.66151460	-1.55384922	-3.8133895	-6.0844011	-9.6185745	-1.0375530
1.75	-0.0010537	-0.0093550	-0.0362987	-0.06175949	-0.13096344	-0.30377450	-0.6008761	-1.3660871	-2.132980	-29.911707	-29.580613
2.0	-0.0048710	-0.0242772	-0.15976002	-0.67526800	-1.1857537	-1.7629444	-1.5739383	-2.4178409	-8.8951165	-23.089665	-66.440668
2.25	-0.00948710	-0.04551416	-0.11399537	-0.6357897	-1.2050310	-1.2602341	-6.5337028	-8.0489232	-3.7724628	-11.930314	-90.485486
2.5	-0.0121286	-0.0903032	-0.58722198	-2.0805242	-2.68104699	-2.2562781	-1.9977891	-10.775772	-12.903440	-2.4449132	-104.76710
2.75	-0.0166070	-0.12329244	-0.7807712	-3.1155745	-2.9399973	-5.2553220	-5.5061230	-5.0853878	-11.451928	-15.280815	-107.34964
3.0	-0.012557	-0.0179008	-0.1133820	-3.6380286	-3.8632976	-5.8752438	-5.6811908	-2.3438196	-10.107103	-23.729279	-98.161884
3.25	-0.10582520	-0.7344000	-2.1846833	-8.6280367	-2.206036	-3.9912150	-2.2500335	-9.6809526	-3.8233130	-22.911853	-81.159650
3.5	-0.25381004	-0.1582530	-0.4342970	-3.2565081	-3.2002607	-3.5554849	-1.1675823				
3.75	-0.0146696	-0.57329224	-2.8813111	-2.2681751							
4.0	-0.05917498	-2.0007059	-1.604251	-1.7443953							
4.25	-0.77949740	-1.1336259	-1.6577770								
4.5	-1.5880756	-1.6169872	-1.2876061								
4.75	-1.1150421	-2.5211396	-1.3776131								
5.0	-1.600803	-1.6000659									
5.25	-1.3060680	-1.6115549									
5.5	-1.7069596	-0.81637904									
5.75	-2.1744354	-0.9496085									
6.0	-3.3429061	-3.4224865									

(r)  $I_9''$ 

$\frac{k_1}{k_2}$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00172104	0.00031552	0.00260981	0.01575271	0.04990728	0.13905562	0.41344770	1.4527999	3.0492245	7.1240304	51.190329
1.0	-0.00140293	0.00045909	0.00353927	0.02480295	0.06616501	0.18339841	0.55035570	1.9350668	4.0631811	9.8888012	68.249152
1.25	-0.00129224	0.00022677	0.00260041	0.02346783	0.06844764	0.20330662	0.62348660	2.3410992	4.9784851	12.231933	85.129968
1.5	-0.00136138	-0.00316001	-0.0152005	-0.05474237	-0.07928228	-0.05861226	-0.22218630	-1.8433449	-4.7051554	-13.021407	-99.772283
1.75	-0.00261159	-0.01001957	-0.0452047	-0.31762973	-0.64347920	-1.2133516	-2.0290746	-2.2716034	-5.8561940	-6.9063604	-103.75368
2.0	-0.00729126	-0.01766863	-0.4371949	-1.1503521	-1.57316310	-1.7736222	-4.1441294	-9.2230307	-11.440652	-9.0445167	-88.000534
2.25	-0.00751775	-0.0561386	-0.3365225	-1.2612863	-1.9399446	-2.1407062	-1.0970430	-7.9296810	-15.360264	-22.230966	-60.243756
2.5	-0.02392201	-0.12430999	-0.31087506	-3.1512850	-1.8418689	-4.0424570	-7.1344303	-2.4428392	-7.4932362	-24.075027	-28.842766
2.75	-0.0152402	-0.09555925	-0.4693493	-2.9804155	-3.6295548	-1.8298110	-4.1440572	-8.6944165	-2.0116519	-19.569027	-6.4016510
3.0	-0.0113673	-0.0857007	-1.3970120	-3.6211113	-2.3135782	-4.9469196	-2.9198747	-10.270226	-11.593320	-7.2311311	-38.168225
3.25	-0.1208493	-0.29231429	-0.66140757	-3.7361317	-3.2266886	-1.4227306	-5.9456852	-5.0075916	-14.775811	-6.0903198	-69.762724
3.5	-0.0941127	-0.2482966	-2.7900027	-0.94203366							
3.75	-0.3982269	-1.4737458	-0.9761848	-2.5906650							
4.0	-1.38392726	-1.2334814	-2.3925852	-0.78115230							
4.25	-0.30320459	-2.0706149	-1.7743587	-2.340963							
4.5	-1.1028718	-2.0778495	-1.2181446								
4.75	-0.8745470	-5.1522062	-0.69261724								
5.0	-0.63493018	-2.2831792									
5.25	-1.9729956	-0.9392285									
5.5	-1.3997486	-1.0341715									
5.75	-0.80112101	-0.4488955									
6.0	-2.2691089	-1.2970906									

TABLE VII.- VALUES OF THE INTEGRAL  $I_1$  FOR  $nB = 12$ (a)  $I_1'$ 

$K_2$	$K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00000424	0.00000621	0.00001482	0.00010621	0.00032760	0.00104587	0.00353305	0.01354458	0.02911699	0.07098013	0.12877715	
1.0	-0.00000470	0.00000206	0.00005380	0.00045373	0.00127033	0.00356799	0.01368820	0.03325191	0.06133919	0.13703002	0.23536978	
1.25	-0.00000215	0.00000342	0.00011770	0.00043700	0.00117704	0.00367144	0.01377845	0.03377845	0.06133919	0.13703002	0.23536978	
1.5	-0.00001490	-0.00002690	-0.00053438	-0.00127235	-0.00356799	-0.01368820	-0.03325191	-0.06133919	-0.13703002	-0.23536978	-0.39505488	
1.75	-0.00007454	-0.00009790	-0.00024332	-0.00065868	-0.00188671	-0.00535266	-0.01517771	-0.03377845	-0.06133919	-0.13703002	-0.23536978	
2.0	-0.00037102	-0.00057179	-0.00125906	-0.00353305	-0.00970937	-0.02308113	-0.05345266	-0.10108693	-0.18137119	-0.2849201	-0.39505488	
2.25	-0.00064000	-0.001259907	-0.00282000	-0.00658671	-0.01517771	-0.03325191	-0.06133919	-0.10108693	-0.18137119	-0.2849201	-0.39505488	
2.5	-0.00377381	-0.00671200	-0.01259906	-0.02820000	-0.05345266	-0.10108693	-0.18137119	-0.2849201	-0.39505488	-0.50515480	-0.59525870	
2.75	-0.01211008	-0.02110008	-0.03669344	-0.05900568	-0.08933548	-0.12880701	-0.17730553	-0.23481921	-0.29795531	-0.36673390	-0.43552966	
3.0	-0.02810553	-0.04896300	-0.07962571	-0.11880701	-0.17730553	-0.23481921	-0.29795531	-0.36673390	-0.43552966	-0.50515480	-0.59525870	
3.25	-0.04679749	-0.07907278	-0.11880701	-0.17730553	-0.23481921	-0.29795531	-0.36673390	-0.43552966	-0.50515480	-0.59525870	-0.68535998	
3.5	-0.06579352	-0.10827271	-0.16204490	-0.22513100	-0.29795531	-0.36673390	-0.43552966	-0.50515480	-0.59525870	-0.68535998	-0.77546126	
3.75	-0.08512528	-0.13948956	-0.20204490	-0.27031216	-0.33190307	-0.39555296	-0.46022401	-0.52589504	-0.59256608	-0.65923712	-0.72590816	
4.0	-0.10477733	-0.1614382	-0.22513100	-0.29795531	-0.36673390	-0.43552966	-0.50515480	-0.59525870	-0.68535998	-0.77546126	-0.86556254	
4.25	-0.12477398	-0.1897398	-0.25959577	-0.33190307	-0.39555296	-0.46022401	-0.52589504	-0.59256608	-0.65923712	-0.72590816	-0.79257920	
4.5	-0.14519882	-0.2193882	-0.29559577	-0.36673390	-0.43552966	-0.50515480	-0.59525870	-0.68535998	-0.77546126	-0.86556254	-0.95566382	
4.75	-0.1666322	-0.2466322	-0.32597258	-0.4058586	-0.48590307	-0.56594748	-0.64599189	-0.72603630	-0.80608071	-0.88612512	-0.96616953	
5.0	-0.1881343	-0.27881343	-0.36673390	-0.4558586	-0.53590307	-0.61594748	-0.69599189	-0.77603630	-0.85608071	-0.93612512	-1.01616953	
5.25	-0.2096372	-0.2996372	-0.3896372	-0.4796372	-0.5596372	-0.6396372	-0.7196372	-0.7996372	-0.8796372	-0.9596372	-1.0396372	
5.5	-0.2311401	-0.3211401	-0.4111401	-0.5011401	-0.5911401	-0.6811401	-0.7711401	-0.8611401	-0.9511401	-1.0411401	-1.1311401	
5.75	-0.2526430	-0.3426430	-0.4326430	-0.5226430	-0.6126430	-0.7026430	-0.7926430	-0.8826430	-0.9726430	-1.0626430	-1.1526430	
6.0	-0.2741459	-0.3641459	-0.4541459	-0.5441459	-0.6341459	-0.7241459	-0.8141459	-0.9041459	-0.9941459	-1.0841459	-1.1741459	

(b)  $I_1''$ 

$K_2$	$K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00000246	-0.00000040	-0.00000449	-0.00000709	-0.00000638	-0.00000464	0.00000119	0.00001138	0.00001771	0.00002776	0.00005019	
1.0	0.00000114	0.00000563	0.00001635	0.00007272	0.00004306	0.00008010	0.00006866	0.00005603	0.00004340	0.00003077	0.00001814	
1.25	-0.00000202	0.00000343	0.00000863	0.00002032	0.00000232	0.00000704	0.00001171	0.00001638	0.00002105	0.00002572	0.00003039	
1.5	-0.00000008	-0.00000215	-0.00000577	-0.00001277	-0.00000337	-0.00000704	-0.00001171	-0.00001638	-0.00002105	-0.00002572	-0.00003039	
1.75	-0.00000449	-0.00001009	-0.00002160	-0.00004774	-0.00001369	-0.00002908	-0.00004537	-0.00006166	-0.00007795	-0.00009424	-0.00011053	
2.0	-0.00002461	-0.00005622	-0.00010028	-0.00018263	-0.00002610	-0.00005610	-0.00009610	-0.00013610	-0.00017610	-0.00021610	-0.00025610	
2.25	-0.00003985	-0.00007163	-0.00012707	-0.00021834	-0.00004096	-0.00008820	-0.00013544	-0.00018268	-0.00022992	-0.00027716	-0.00032440	
2.5	-0.00007128	-0.00012710	-0.00023044	-0.00038206	-0.00008538	-0.00017070	-0.00025602	-0.00034134	-0.00042666	-0.00051198	-0.00059730	
2.75	-0.00009606	-0.00016652	-0.00028652	-0.00045707	-0.00010604	-0.00021209	-0.00031814	-0.00042419	-0.00053024	-0.00063629	-0.00074234	
3.0	-0.00012569	-0.00021700	-0.00035255	-0.00053255	-0.00012780	-0.00025560	-0.00038340	-0.00051120	-0.00063900	-0.00076680	-0.00089460	
3.25	-0.00016888	-0.00028074	-0.00045765	-0.00069881	-0.00016354	-0.00032708	-0.00049062	-0.00065416	-0.00081770	-0.00098124	-0.00114478	
3.5	-0.00021071	-0.00034229	-0.00055862	-0.00083962	-0.00021504	-0.00043008	-0.00064512	-0.00086016	-0.00107520	-0.00129024	-0.00150528	
3.75	-0.00025886	-0.00041823	-0.00066823	-0.00100823	-0.00025527	-0.00051054	-0.00076581	-0.00102108	-0.00127635	-0.00153162	-0.00178689	
4.0	-0.00030701	-0.00049242	-0.00075242	-0.00115242	-0.00030000	-0.00060000	-0.00090000	-0.00120000	-0.00150000	-0.00180000	-0.00210000	
4.25	-0.00035516	-0.00056955	-0.00086955	-0.00132955	-0.00034500	-0.00069000	-0.00103500	-0.00138000	-0.00172500	-0.00207000	-0.00241500	
4.5	-0.00040331	-0.00063709	-0.00097109	-0.00147109	-0.00039000	-0.00078000	-0.00117000	-0.00156000	-0.00195000	-0.00234000	-0.00273000	
4.75	-0.00045146	-0.00070465	-0.00105865	-0.00161865	-0.00043500	-0.00087000	-0.00130500	-0.00174000	-0.00217500	-0.00261000	-0.00304500	
5.0	-0.00050000	-0.00076938	-0.00113876	-0.00170814	-0.00048000	-0.00096000	-0.00144000	-0.00192000	-0.00240000	-0.00288000	-0.00336000	
5.25	-0.00054815	-0.00083253	-0.00124691	-0.00186129	-0.00052500	-0.00105000	-0.00157500	-0.00210000	-0.00262500	-0.00315000	-0.00367500	
5.5	-0.00059630	-0.00089590	-0.00135530	-0.00201970	-0.00057000	-0.00114000	-0.00171000	-0.00228000	-0.00285000	-0.00342000	-0.00399000	
5.75	-0.00064445	-0.00095449	-0.00141449	-0.00212889	-0.00061500	-0.00123000	-0.00184500	-0.00246000	-0.00307500	-0.00369000	-0.00430500	
6.0	-0.00069260	-0.00101300	-0.00147300	-0.00223740	-0.00066000	-0.00132000	-0.00198000	-0.00264000	-0.00330000	-0.00396000	-0.00462000	

TABLE VII.- VALUES OF THE INTEGRAL  $I_2$  FOR  $m = 12$  - Continued(a)  $I_2'$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00000554	0.00000866	0.00002184	0.00018950	0.00067276	0.00258007	0.01058608	0.05295900	0.13601290	0.41698850	1.1938434
1.0	-0.00000648	-0.00003287	-0.0001248	-0.00015918	-0.00059589	-0.00234830	-0.01010995	-0.05203997	-0.13471243	-0.41616120	-1.1909472
1.25	-0.00000001	-0.00004299	-0.00036994	-0.00236692	-0.00537829	-0.01111679	-0.01894348	-0.0824846	-0.2002759	-0.59439420	-1.2982177
1.5	-0.00000913	-0.00021484	-0.00264813	-0.02242686	-0.05871779	-0.11641029	-0.31394112	-0.75532950	-1.6667880	-3.3566941	-7.8822552
1.75	-0.0001583	-0.00151625	-0.01189092	-0.0878968	-0.6221105	-0.0922628	-0.11235812	-1.8213099	-3.3368296	-5.7275255	-8.4002610
2.0	-0.0029339	-0.0213992	-0.0020091	-0.1262961	-0.5896830	1.1552196	2.2551240	2.7217396	1.4815928	-2.3282658	-13.998632
2.25	-0.0001072	-0.0079816	-0.10587679	-0.69114653	-1.2776672	-1.6587517	-1.6802408	3.0864247	5.0173344	3.9630612	-13.718894
2.5	-0.0088716	-0.0418302	-0.35085704	-1.0637303	-9.0757611	-5.6917904	-3.0611467	-2.1123146	1.6523638	5.1759180	-11.783418
2.75	-0.01156159	-0.13228208	-0.59512430	-3.2231133	1.1636291	2.7817797	1.8747711	-3.2378274	-1.9751008	4.2218911	-7.3376686
3.0	-0.0089886	-0.3009955	-2.350469	-1.009785	-2.5111495	-9.4700374	2.3686699	-1.671938	-3.8816893	6.6803510	-1.5810849
3.25	-0.0152169	-0.15893515	-1.3739035	-1.1222882	0.93208260	5.2721146	1.0400979	1.8090019	-2.9735653	-3.6900559	2.8617156
3.5	-0.09606364	-0.8627224	-2.1316954	-1.7833773	1.2566912	1.6163427					
4.0	-0.3219477	-1.9958986	-1.7212268	1.4504451	2.8458778						
4.25	-1.4591059	-2.0910289	8.1577292								
4.5	-1.4642563	9.0807300	1.8306090	-9.0279655							
4.75	-1.3765551	-0.8933386									
5.0	1.7556366	-0.645291									
5.25	-1.8961537	6.2916178									
5.5	1.7097468	-5.2790407									
5.75	-1.2156959	-1.1914443									

(a)  $I_2''$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.925	.95	.985
0.75	0.00000323	0.00000194	0.00008842	0.00036500	0.00130192	0.00445801	0.01630053	0.06907054	0.15831891	0.41681960	2.9718028
1.0	-0.0000234	-0.0001532	-0.0011670	-0.0107168	-0.0328368	-0.10131234	-0.3146199	-1.1218346	-2.663215	-7.0875130	-4.5184733
1.25	-0.0000395	-0.0003021	-0.0044219	-0.0125277	-0.0193206	-0.03433121	-0.10163668	-0.33056349	-0.8583152	-1.4124662	-7.1810736
1.5	-0.0001280	-0.0024216	-0.0182493	-0.0704750	-0.0722951	-0.1339725	-0.18418048	-0.59410910	-1.8561155	-2.7747345	-12.129310
1.75	-0.0007936	-0.0046665	-0.0827912	-0.07982125	-0.2089350	-0.8788279	-0.98114070	-1.4857134	-1.2850969	-2.2166290	-13.463493
2.0	-0.0036326	-0.00587806	-0.05111752	-0.29036201	-0.40410197	-0.3682165	-0.39922112	-3.0542720	-5.1969010	-6.8436037	-4.7043240
2.25	-0.016368	-0.01939486	-0.10275334	-0.09973007	-0.33960168	1.3543566	3.4419837	1.5175001	0.92450755	-4.1055472	-6.6799740
2.5	-0.049219	-0.0318477	-0.2364392	-0.83839349	-2.0200116	-3.0703323	-1.8960926	2.5071699	3.2185843	-1.3675883	-7.8533102
2.75	-0.0622100	-0.0581184	-0.37839876	2.0810487	2.5815700	1.0652699	-1.8997421	3.2664566	4.2365132	4.2196710	-10.608503
3.0	-0.0110553	-0.12321984	-1.1044159	-2.1893861	-0.69914936	1.8028084	-0.80339017	-3.6514125	-1.3666136	4.9140122	-13.092595
3.25	-0.0153576	-0.0827050	1.7565596	6.0412452	-1.2125963	0.4236700	3.0012061	-2.0270469	-2.5868977	4.2988004	-11.963197
3.5	-0.08916138	-0.82273576	-1.7361867	0.77104910		-2.16929379	0.66603173				
3.75	-0.0822430	1.21289666	8.0116360	-0.8784292							
4.0	-0.38052083	-1.3374716	-4.2147149	-1.3621816							
4.25	-0.7446701	1.0103988	-0.88534933	1.8701449							
4.5	-0.7238144	-0.27363480	1.0358572								
4.75	-0.71170675	-0.54108708	1.2078541								
5.0	-0.5447106	0.96169860									
5.25	-0.14368325	-0.67801775									
5.5	-0.1019225	-0.20033683									
5.75	-0.77770850	1.1530193									
6.0	1.3325525	-1.5701334									

TABLE VII.- VALUES OF THE INTEGRAL  $I_p$  FOR  $m = 12$  - Continued(a)  $I_3'$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.95	.95	.985
0.75	0.00150244	0.00000688	-0.00003785	-0.00043850	-0.00116686	-0.00243809	0.00015630	0.07164240	0.31259540	1.5230303	36.589500
1.0	-0.00187060	-0.00002358	-0.00022902	-0.00237917	-0.00737842	-0.02280195	-0.07110260	-0.21608620	-0.32942620	-1.11738420	-25.505754
1.25	-0.00193758	-0.00008804	-0.00099385	-0.0026678	-0.02752227	-0.08207686	-0.25380360	-0.85032020	-1.6861938	-3.1208991	-8.4090400
1.5	-0.00170727	-0.00089023	-0.01729294	-0.01729294	-0.01729294	-0.01729294	-0.01729294	-0.01729294	-0.01729294	-0.01729294	-0.01729294
1.75	-0.00132951	-0.00185667	-0.01849417	-0.01709875	-0.01369753	-0.0066390	-0.00219610	-0.00075341	-0.00028759	-0.000153981	-0.000079508
2.0	-0.00103582	-0.00590617	-0.02937621	-0.02828898	-0.0222114	-0.01590343	-0.00930031	-0.00511847	-0.002831	-0.00153981	-0.00082294
2.25	-0.00057541	-0.0141173	-0.07131660	-0.0487626	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294
2.5	-0.0019674	-0.0352050	-0.38754837	-0.15785286	-0.07015127	-0.03785270	-0.019956990	-0.00931689	-0.0048884	-0.0023264	-0.00122294
2.75	-0.0118300	-0.13984672	-0.78796674	-0.27899795	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294
3.0	-0.03337407	-0.29913763	-0.80065600	-0.16890336	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294
3.25	-0.06907775	-0.41934840	-0.0172975	-0.0172975	-0.0172975	-0.0172975	-0.0172975	-0.0172975	-0.0172975	-0.0172975	-0.0172975
3.5	-0.11828647	-0.3243884	-0.15057553	-0.04612213	-0.02752227	-0.01590343	-0.00930031	-0.00511847	-0.002831	-0.00153981	-0.00082294
3.75	-0.14479126	-0.14385367	-0.2575285	-0.0914392	-0.06714392	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264
4.0	-0.10233877	-0.96791690	-2.1641889	-0.0567274	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294
4.25	-0.06666575	1.8572849	-0.36117084	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294	-0.00082294
4.5	-0.3986983	-2.3383617	1.3126966	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294	-0.00082294
4.75	-0.7949887	-2.0389869	-1.4149548	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294	-0.00082294
5.0	-1.4184852	-0.99640276	-0.26457552	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294	-0.00082294
5.25	-1.8404949	-0.0324024	1.0141181	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294	-0.00082294
5.5	-2.0324024	-0.8100647	-0.11185914	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294	-0.00082294
5.75	-1.8213323	-0.11185914	-0.11185914	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294	-0.00082294
6.0	-1.2411918	-0.11185914	-0.11185914	-0.0399412	-0.0295422	-0.018911689	-0.00931689	-0.0048884	-0.0023264	-0.00122294	-0.00082294

(c)  $I_3''$ 

$K_2 \backslash K_1$	.4	.5	.6	.7	.75	.8	.85	.9	.95	.95	.985
0.75	0.00125279	0.00000797	0.00012161	0.00167876	0.00699266	0.02298114	0.09516060	0.47705170	1.2246394	3.7535859	10.446386
1.0	-0.0057106	-0.00020884	-0.0018750	-0.0224934	-0.08097114	-0.3051957	-1.2657270	-4.63242010	-1.6319507	-5.0035151	-53.926217
1.25	-0.00198492	-0.00027888	-0.0009187	-0.0074734	-0.02524758	-0.02721702	-0.13474900	-0.75591870	-1.9711695	-6.1581348	-87.255203
1.5	-0.00090492	-0.0042745	-0.0437665	-0.01178610	-0.0177335	-0.0177335	-0.0177335	-0.0177335	-0.0177335	-0.0177335	-0.0177335
1.75	-0.00144796	-0.0061844	-0.0013657	-0.05526890	-0.02948241	-0.06722710	-0.16679230	-0.4675294	-1.06911441	-2.3391347	-77.014348
2.0	-0.0025777	-0.0465679	-0.0589361	-0.08083175	-0.07707330	-0.1855179	-0.57337470	-1.4641315	-3.123822	-6.593731	-31.573977
2.25	-0.00358134	-0.02343824	-0.16243562	-0.11530318	-0.05880628	-0.1667179	-0.5217023	-1.4641315	-3.123822	-6.593731	-31.573977
2.5	-0.00704505	-0.05183140	-0.15351833	-0.28551655	-0.16907686	-0.432262	-1.1371499	-2.3678111	-5.057947	-11.4780143	-49.636384
2.75	-0.0145276	-0.0973683	-0.26436151	-0.5840512	-0.4437669	-1.3653015	-3.439517	-7.825315	-14.825315	-27.371915	-77.338006
3.0	-0.1118578	-0.06517404	-1.1719209	-2.9154444	-0.70701394	-1.4501840	-3.3824060	-8.9400060	-19.0863240	-37.751660	-91.881141
3.25	-0.0899278	-0.37522768	-2.0807857	-5.7422638	-2.5706113	-5.3340259	-6.0384738	-6.6020738	-9.1252255	-14.742118	-89.289720
3.5	-0.07226674	-0.86846324	-2.1250818	-1.9314278	-1.2041794	-0.8488773	-0.6020738	-0.4020738	-0.2020738	-0.1020738	-0.0520738
3.75	-0.19949303	-1.3719109	-0.90669322	-1.2041794	-0.8488773	-0.6020738	-0.4020738	-0.2020738	-0.1020738	-0.0520738	-0.0220738
4.0	-0.39573402	-1.5723359	-0.8030610	-1.8492310	-1.2041794	-0.8488773	-0.6020738	-0.4020738	-0.2020738	-0.1020738	-0.0520738
4.25	-0.62907197	-1.2091139	-1.5014113	-2.7130604	-1.2041794	-0.8488773	-0.6020738	-0.4020738	-0.2020738	-0.1020738	-0.0520738
4.5	-0.8488773	-0.30189827	-0.3382230	-1.4390602	-1.2041794	-0.8488773	-0.6020738	-0.4020738	-0.2020738	-0.1020738	-0.0520738
4.75	-0.7841786	-0.74625359	-1.4390602	-1.2041794	-0.8488773	-0.6020738	-0.4020738	-0.2020738	-0.1020738	-0.0520738	-0.0220738
5.0	-0.69508254	1.3230712	-1.2041794	-0.8488773	-0.6020738	-0.4020738	-0.2020738	-0.1020738	-0.0520738	-0.0220738	-0.0120738
5.25	-0.28299780	-1.0036432	-0.0556318	-0.0556318	-0.0556318	-0.0556318	-0.0556318	-0.0556318	-0.0556318	-0.0556318	-0.0556318
5.5	-0.4044096	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952
5.75	-1.0658367	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952
6.0	-1.5035855	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952	-1.3099952

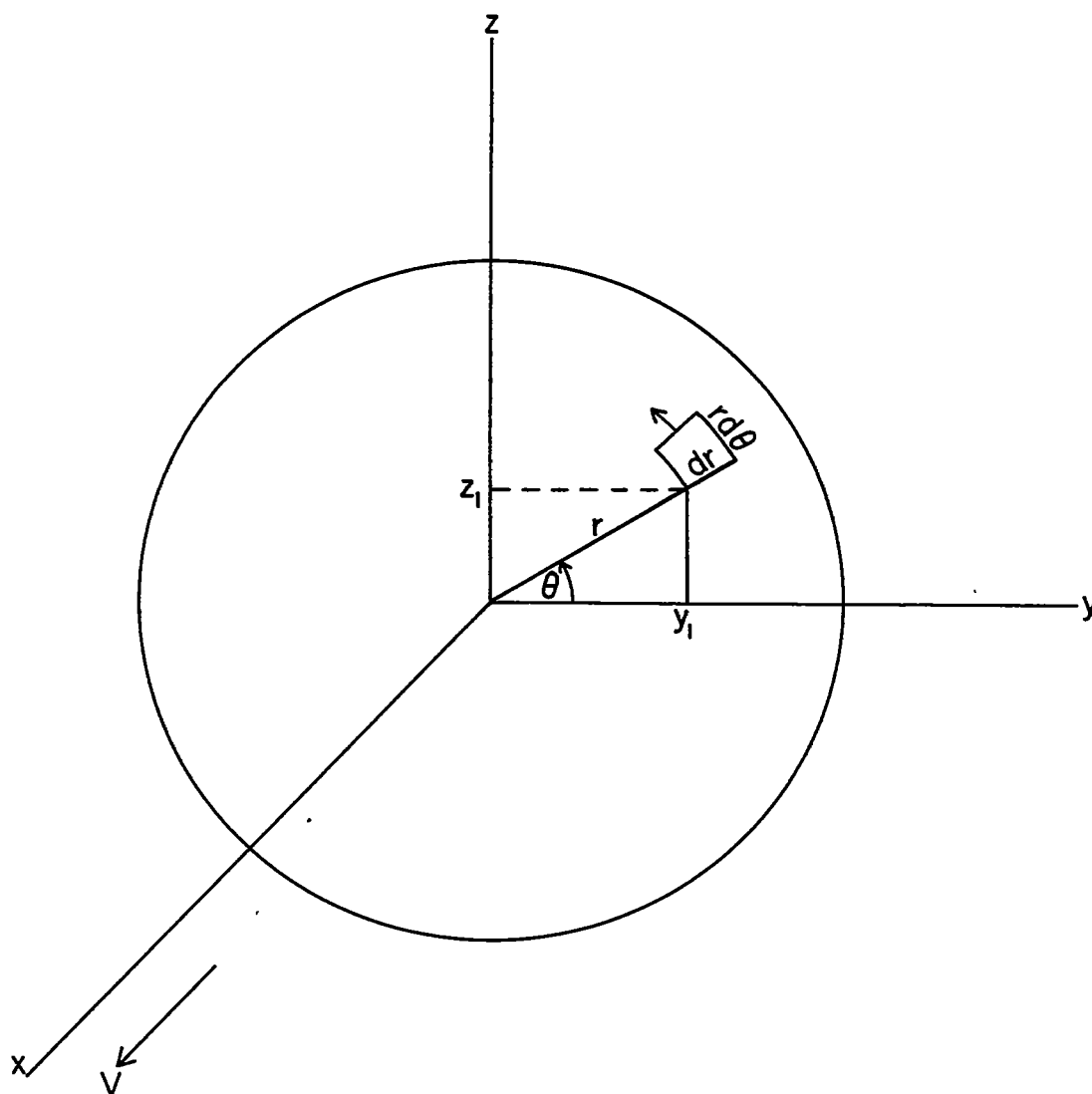


Figure 1.- Propeller disk and coordinate system.



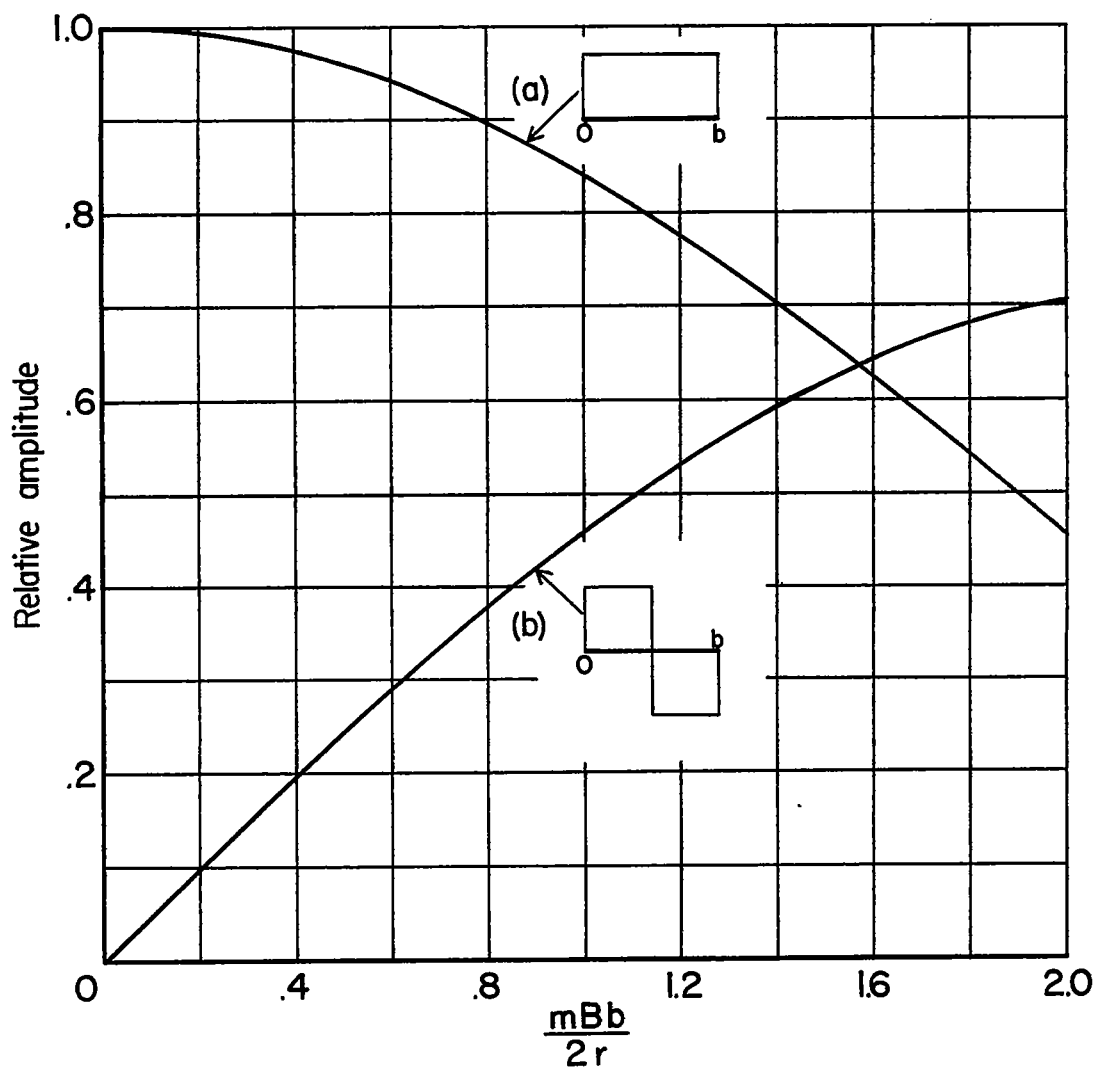
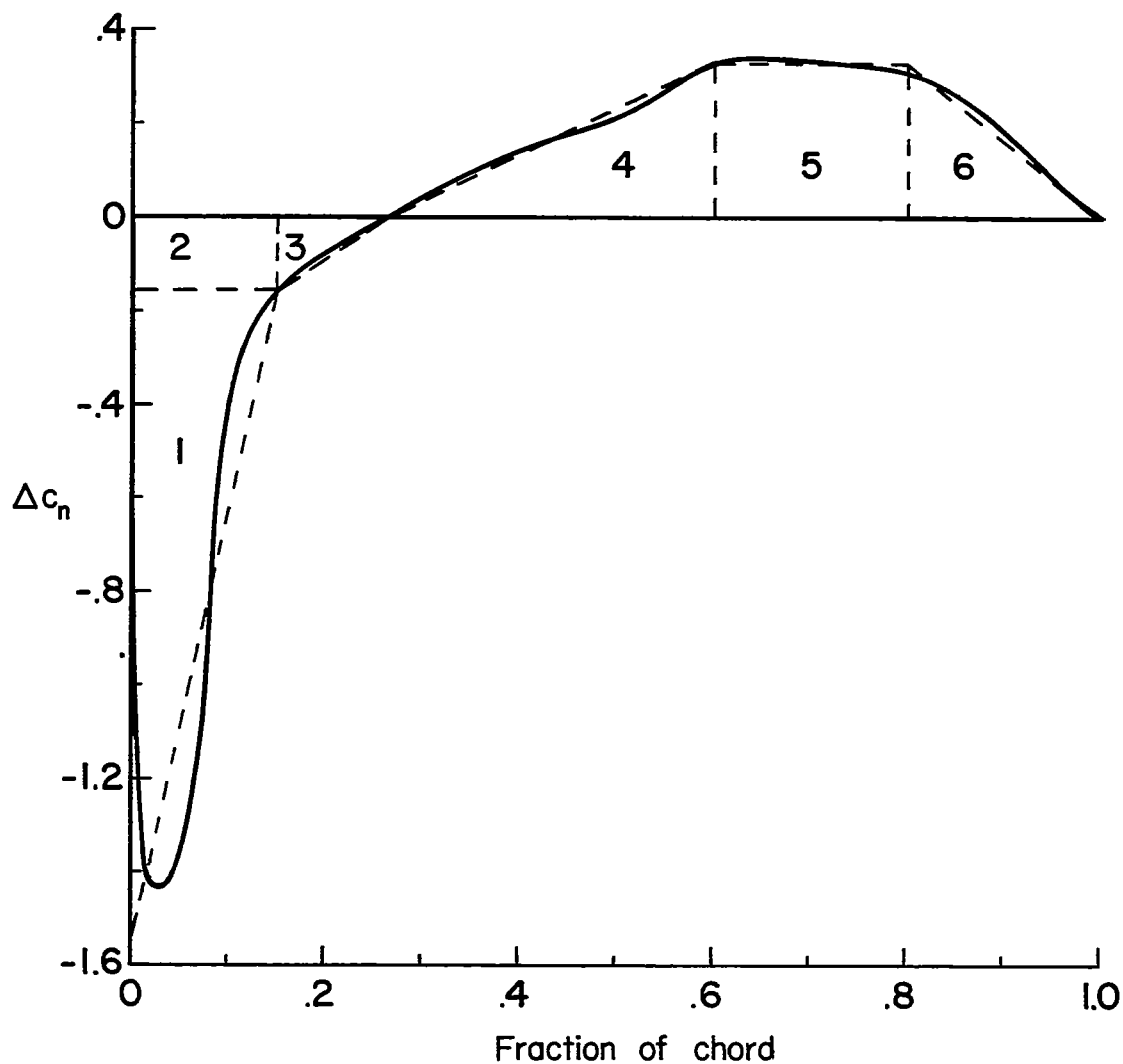
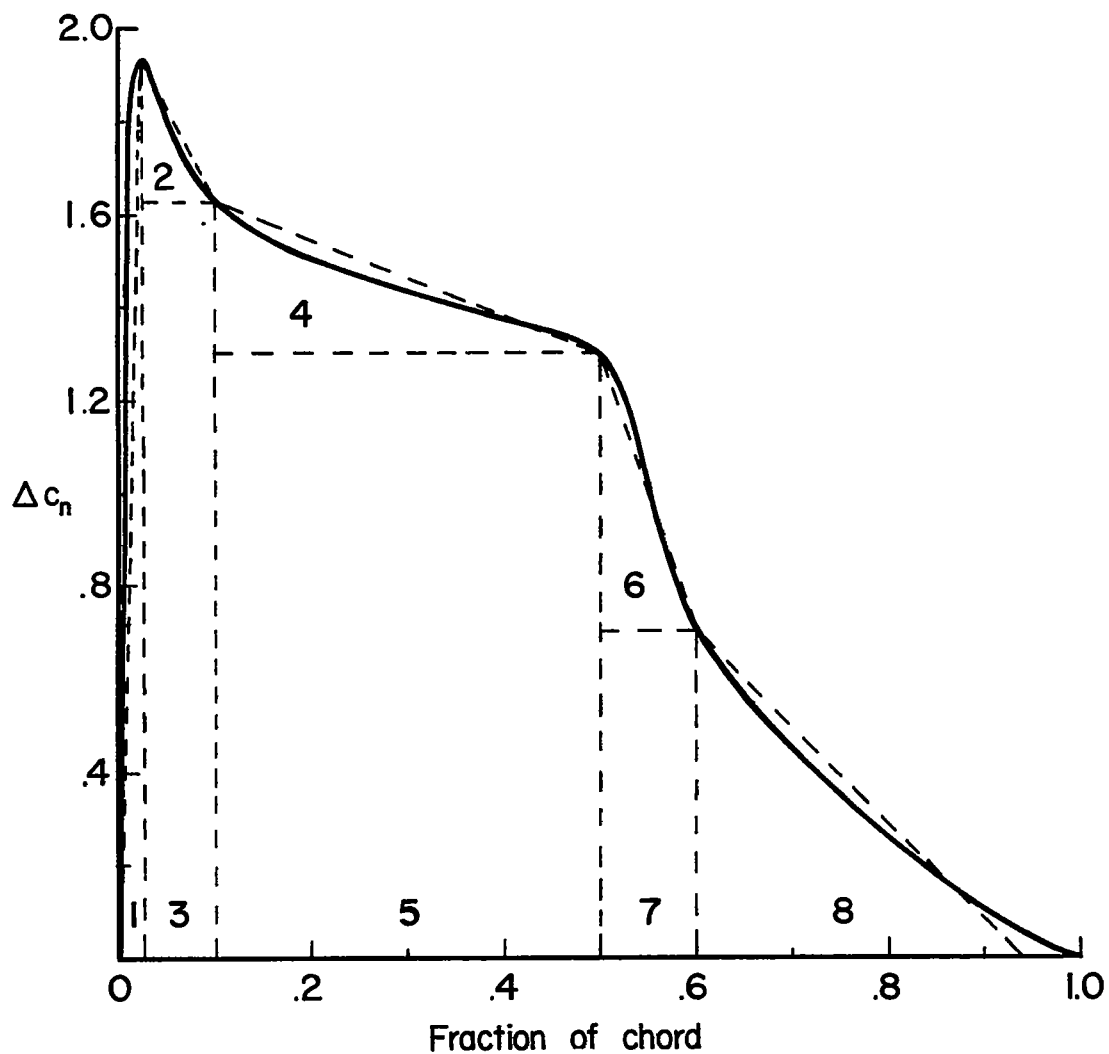


Figure 2.- Relative amplitude functions, (a)  $\frac{2r}{mBb} \sin \frac{mBb}{2r}$  and  
 (b)  $\frac{4r}{mBb} \sin^2 \frac{mBb}{4r}$ , plotted as functions of  $\frac{mBb}{2r}$ .



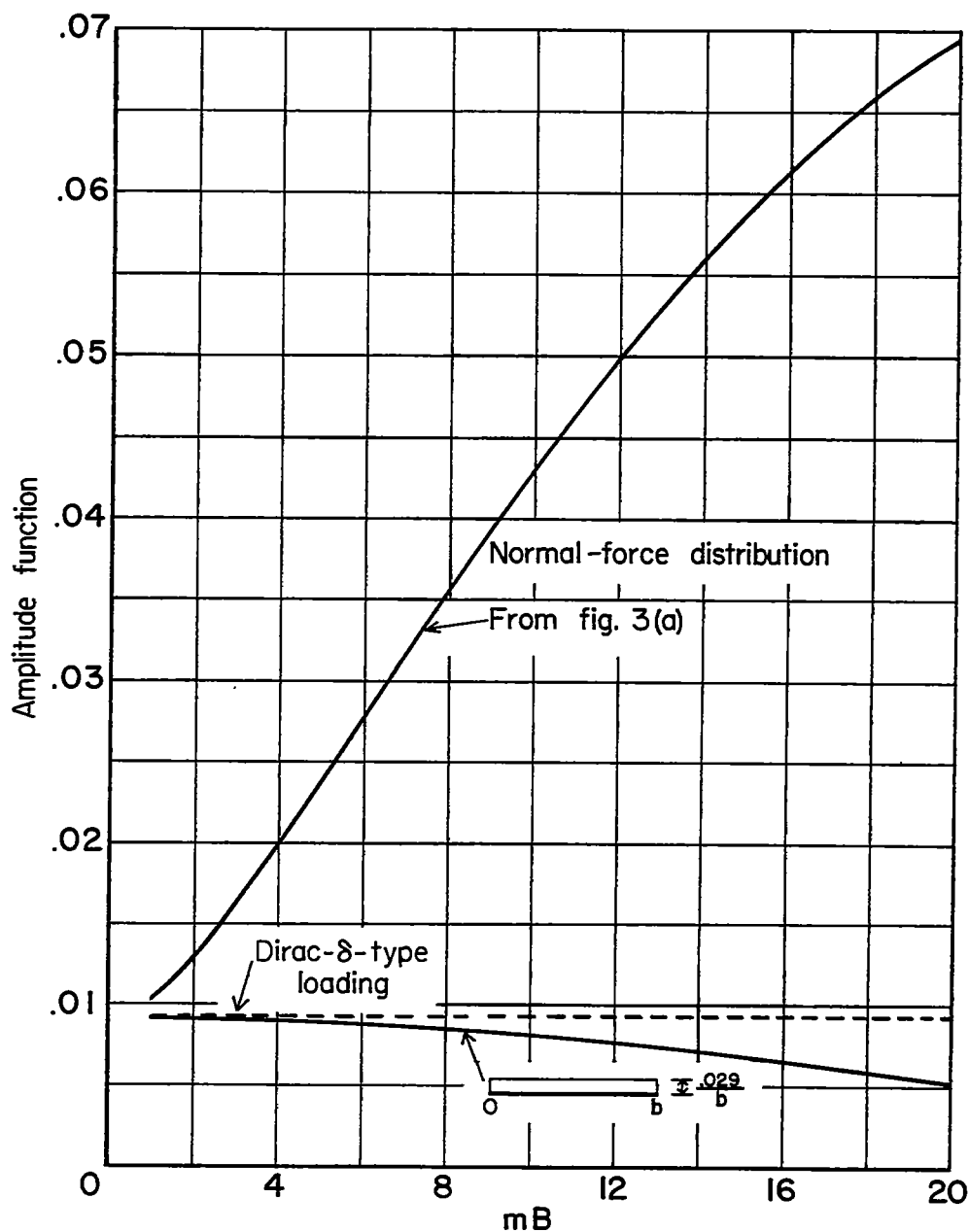
(a)  $c_n = 0.029$ ;  $J = 2.55$ ;  $M_r = 0.8$ ;  $n = 1,500$  rpm;  $\beta_{0.75R} = 45^\circ$ .

Figure 3.- Chordwise normal-force distribution at  $0.78R$  for two sets of operating conditions (ref. 4). (Numbers indicate rectangles and triangles used for the analysis.)



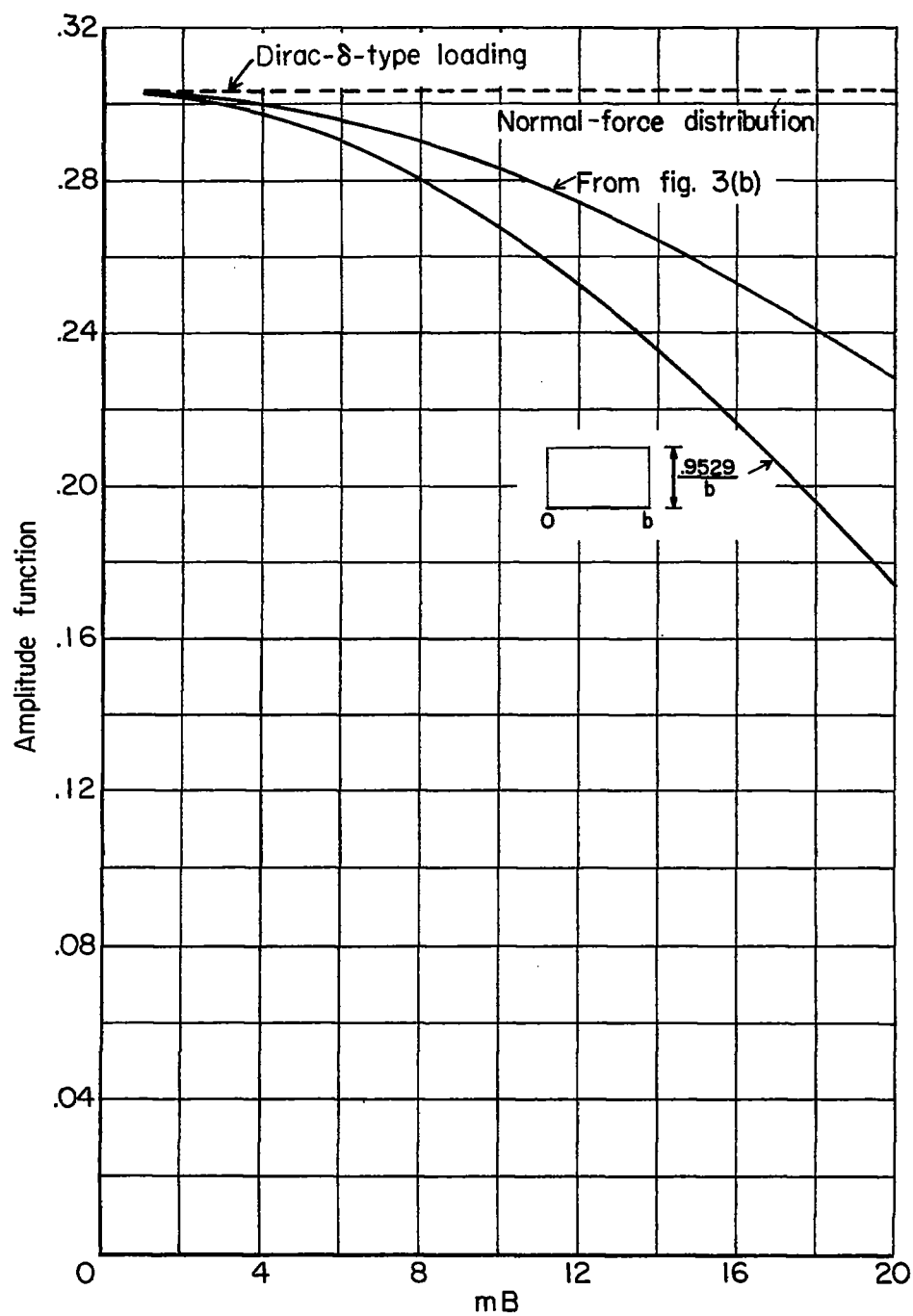
(b)  $c_n = 0.9529$ ;  $J = 1.009$ ;  $M_r = 0.789$ ;  $n = 2,000$  rpm;  $\beta_{0.75R} = 30^\circ$ .

Figure 3.- Concluded.



(a)  $c_n = 0.029$ ;  $J = 2.55$ ;  $M_r = 0.8$ ;  $n = 1,500$  rpm;  $\beta_{0.75R} = 45^\circ$ .

Figure 4.- Amplitude functions associated with the normal-force distributions in figure 3 plotted as functions of  $mB$  compared with amplitude functions associated with uniform distributions of normal force.  $b/r = 0.1715$ .



(b)  $c_n = 0.9529$ ;  $J = 1.009$ ;  $M_r = 0.789$ ;  $n = 2,000$  rpm;  $\beta_{0.75R} = 30^\circ$ .

Figure 4.- Concluded.

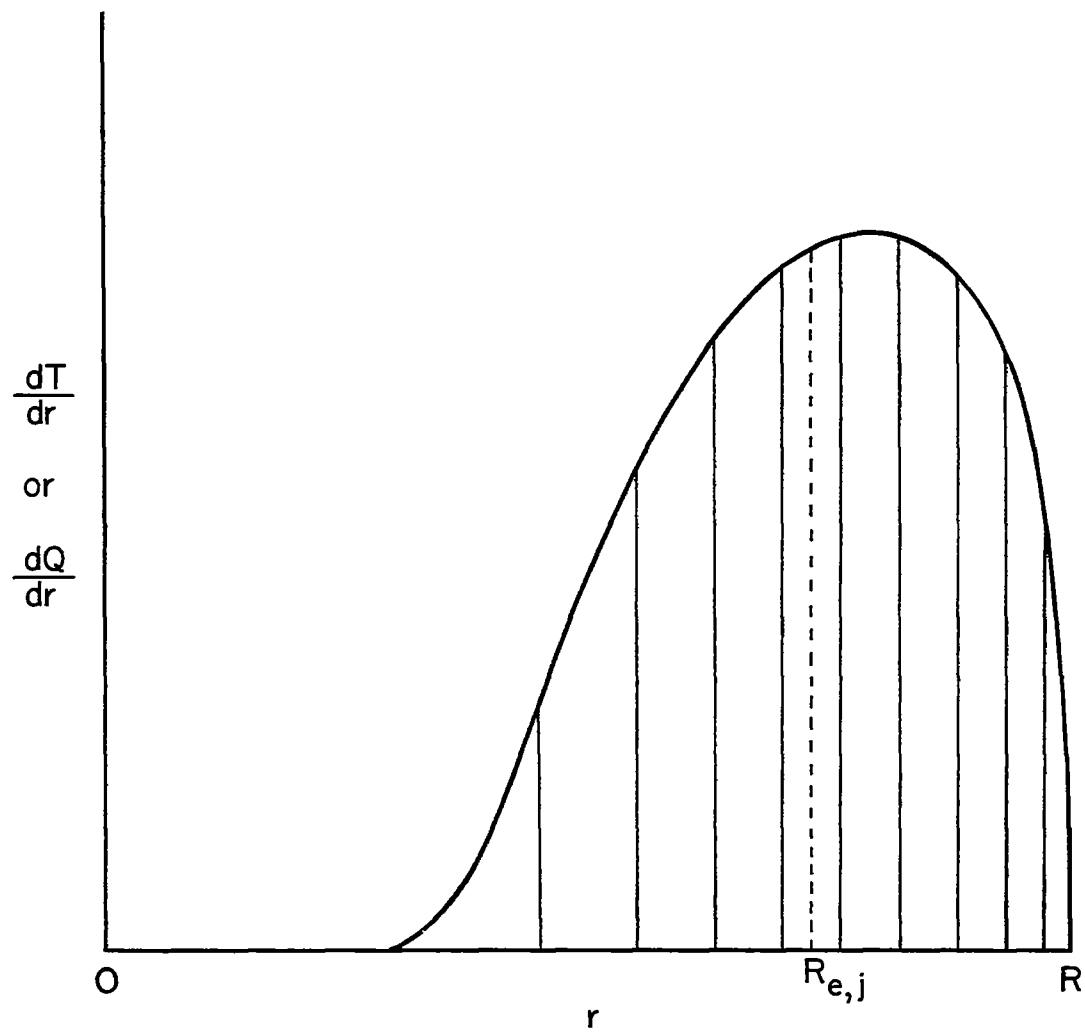
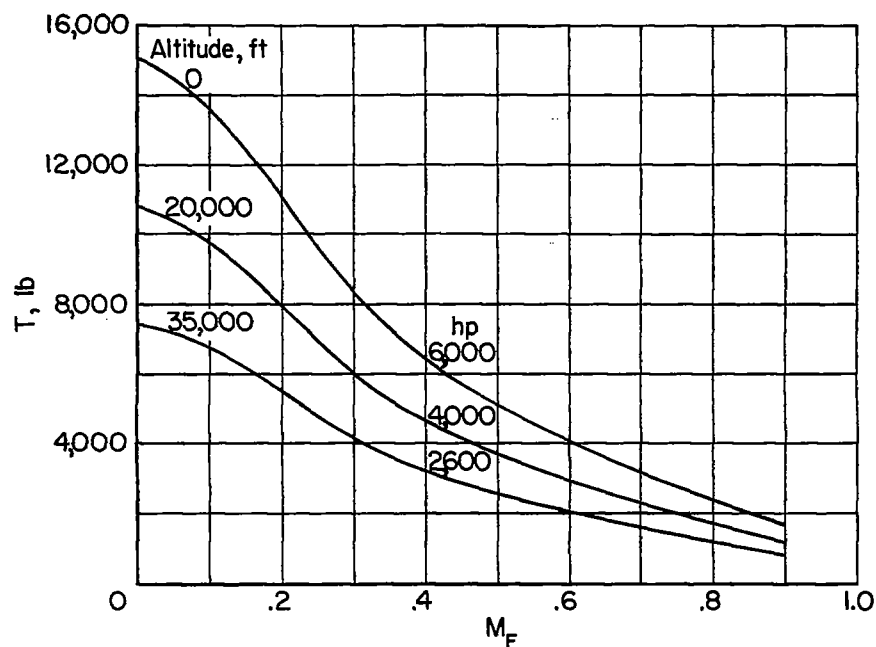
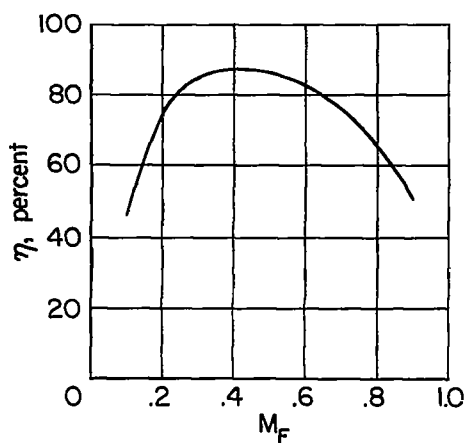


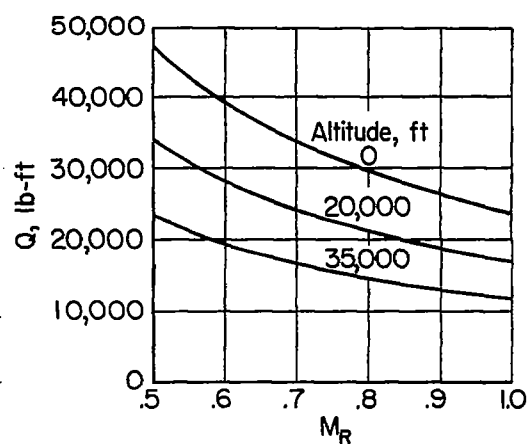
Figure 5.- Sketch of radial thrust or torque distribution.



(a) Thrust.



(b) Efficiency.



(c) Torque.

Figure 6.- Efficiency, total thrust, and total torque for various operating conditions for a 16-foot-diameter propeller.

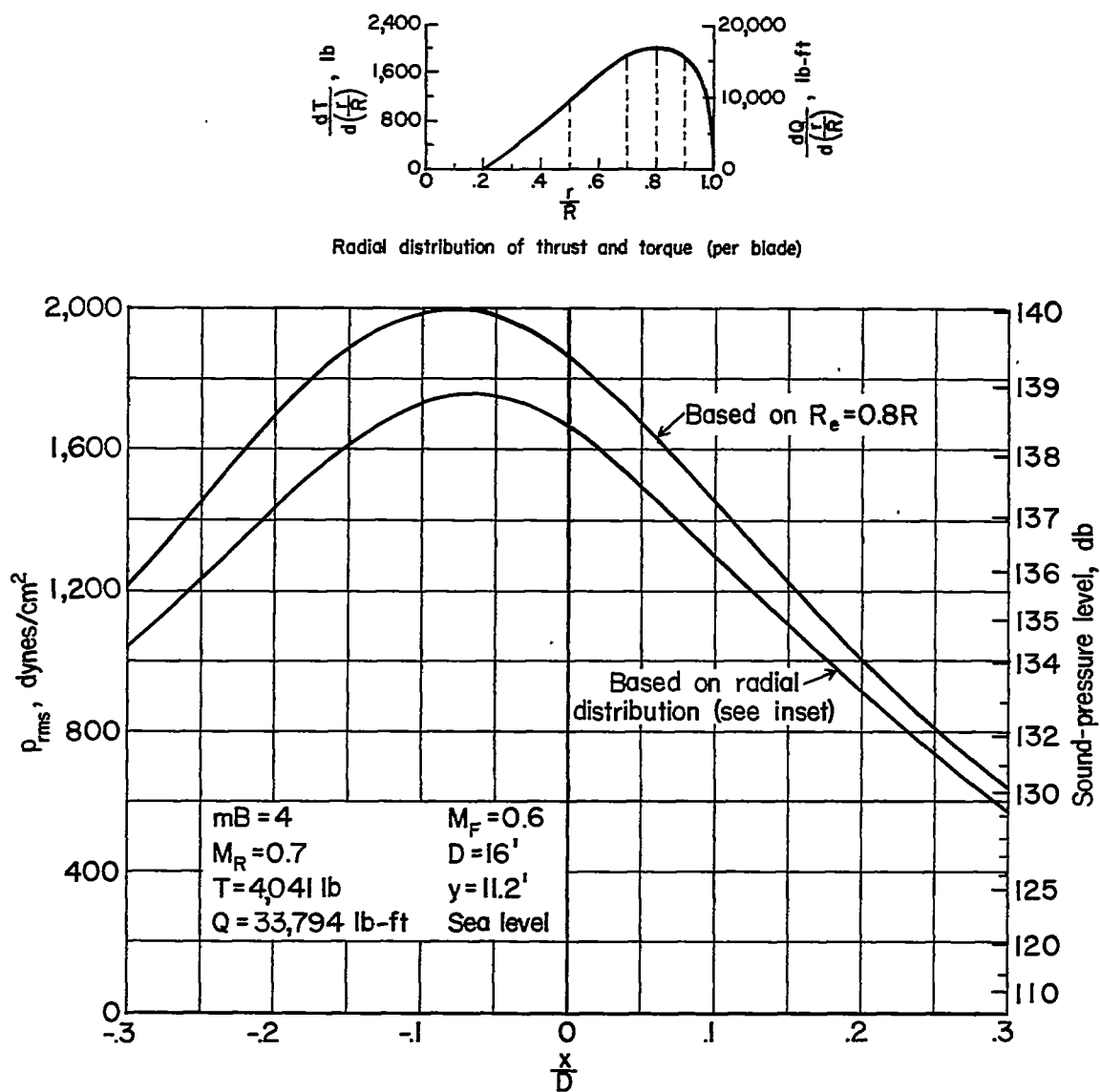


Figure 7.- Comparison of sound-pressure calculations for forces concentrated at an effective radius with calculations for forces distributed along the propeller radius.



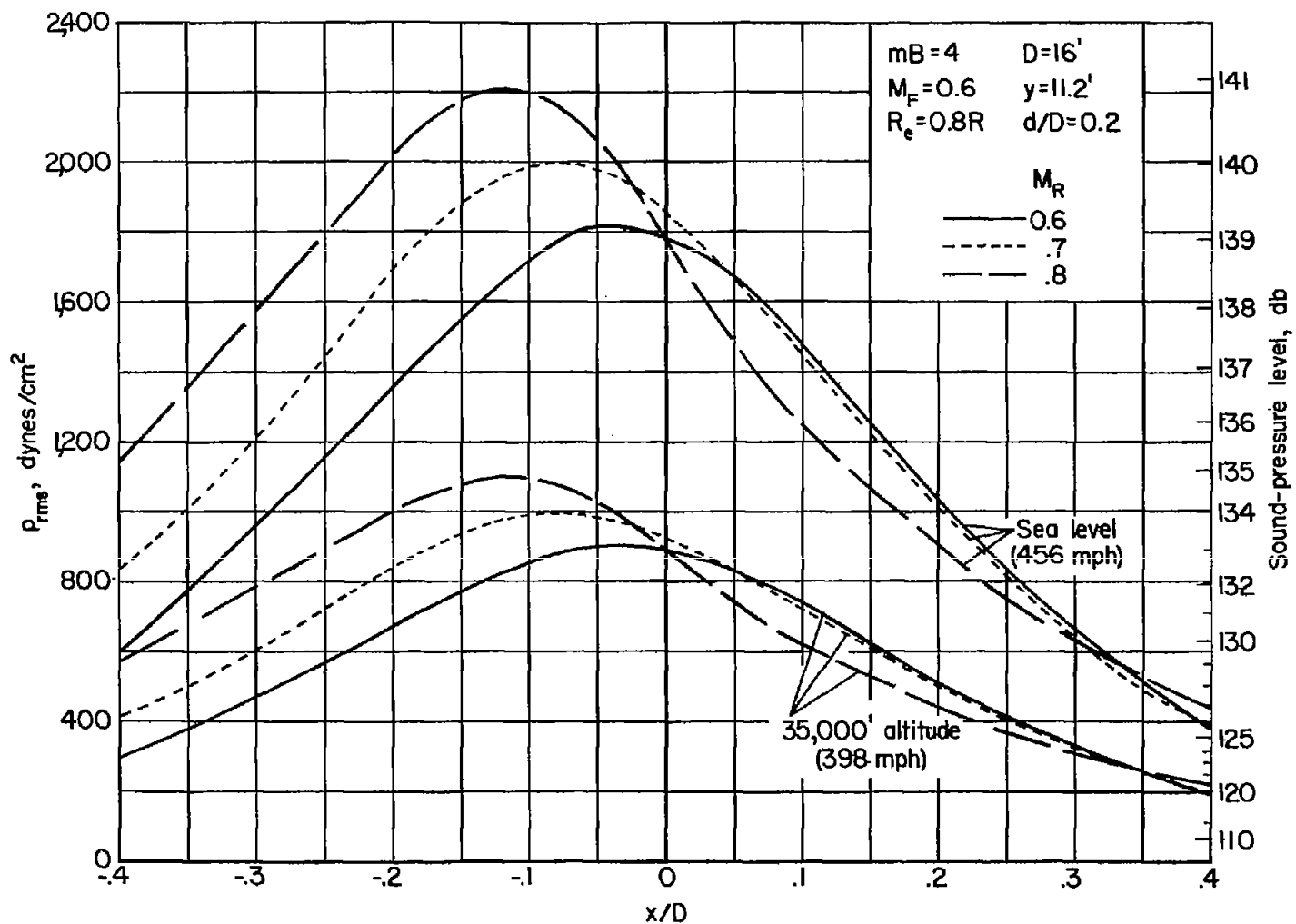


Figure 8.- Distribution of the root-mean-square pressures for the first harmonic of a four-blade 16-foot-diameter propeller at two altitudes and several tip Mach numbers.

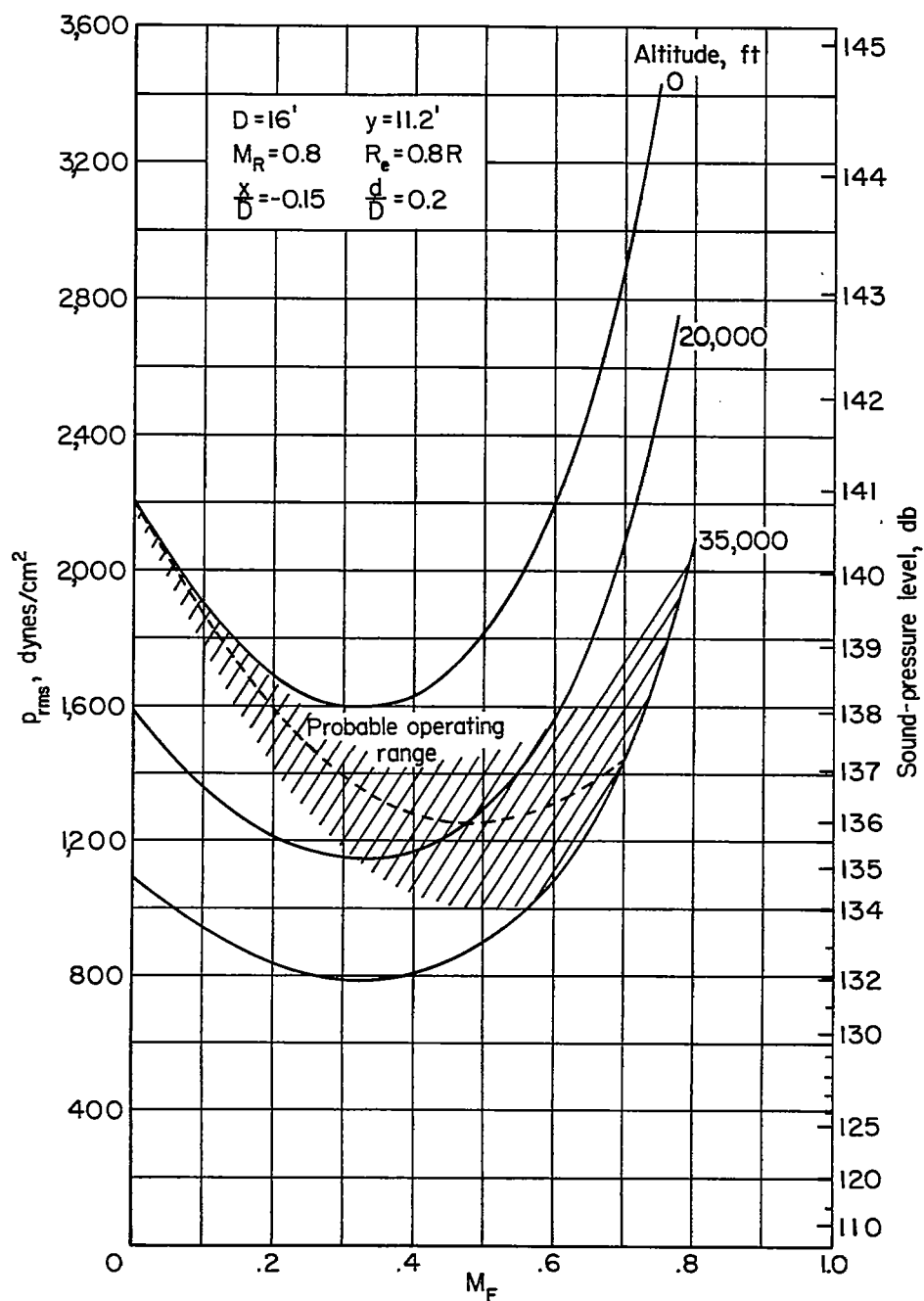
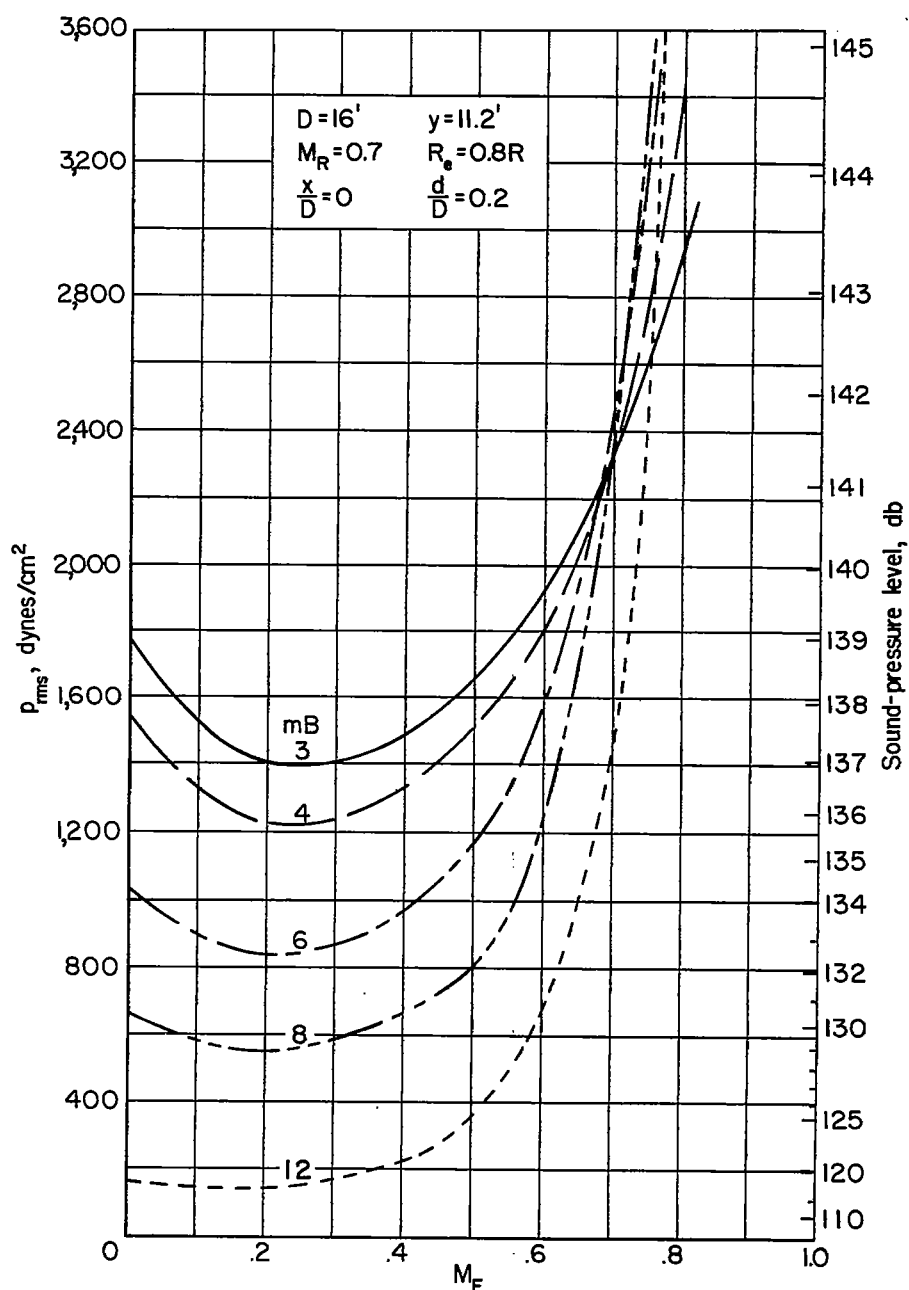
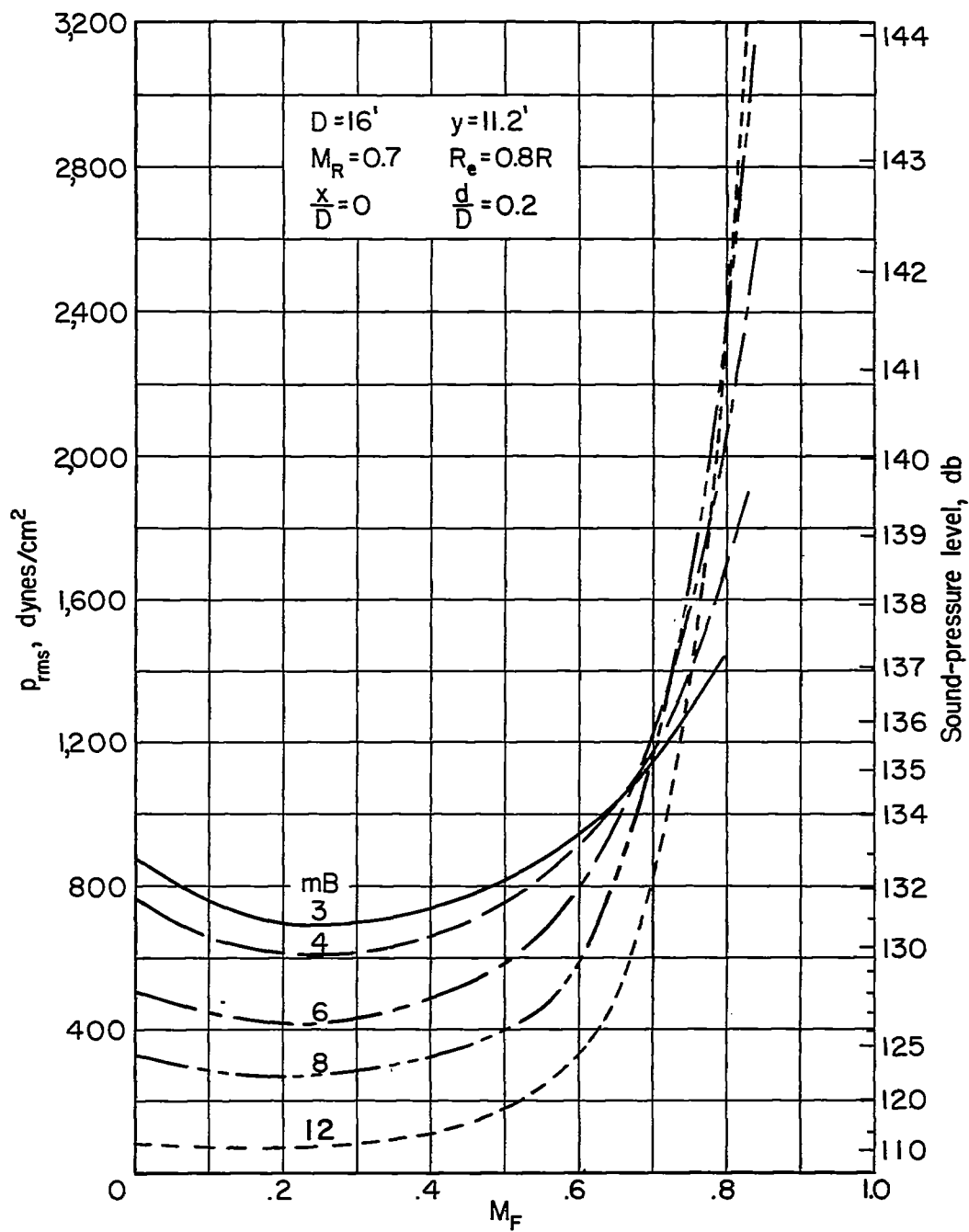


Figure 9.- Calculated root-mean-square pressures associated with first harmonic of a four-blade 16-foot-diameter propeller at a point 0.15D behind the plane of rotation as functions of flight Mach number for three different altitudes.



(a) Sea-level altitude.

Figure 10.- Calculated root-mean-square pressures for various values of  $mB$  in the plane of rotation for 16-foot-diameter propellers as functions of flight Mach number for two different altitudes.



(b) 35,000-foot altitude.

Figure 10.- Concluded.

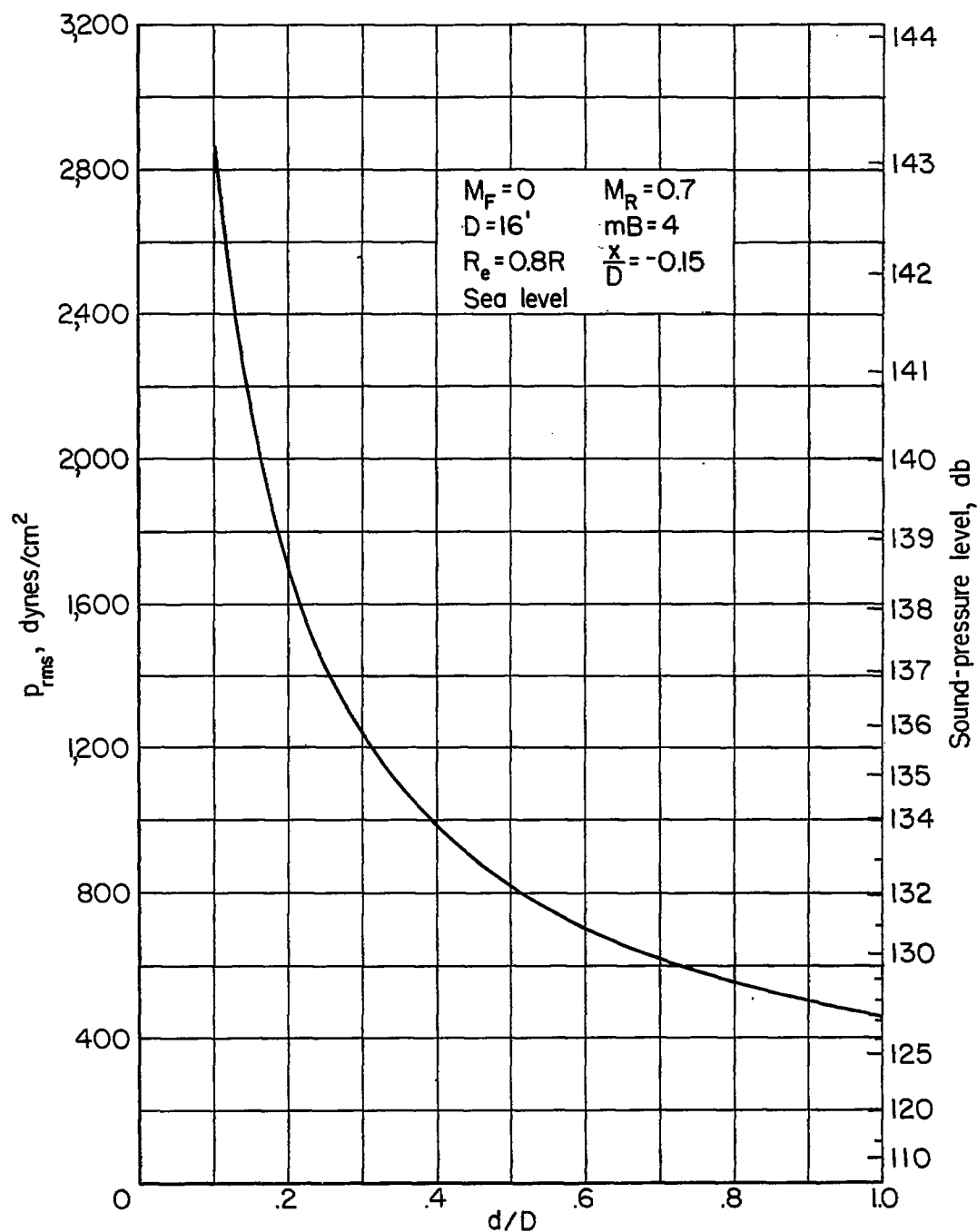


Figure 11.- Calculated root-mean-square pressure as a function of distance from propeller tip along a line  $0.15D$  behind the plane of rotation for a four-blade 16-foot-diameter propeller operating at sea-level conditions.